Capítulo 13 - Troca de calor por radiação entre duas superfícies

-Vamos considerar apenas a situação em que o meio entre as duas superfícies é não participante (não absorve, não emite e não distribui radiação, e portanto não tem nenhum efeito na troca de calor entre as superfícies).

-Exs.: vácuo, maioria dos gases

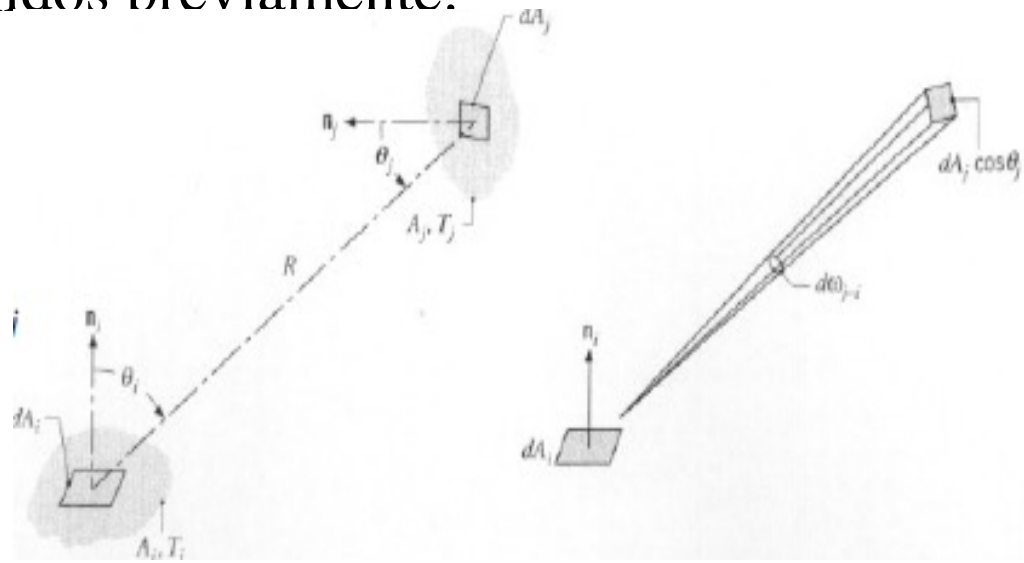
-Fator de forma:

- fração de radiação que deixa uma superfície i (J_i) e é interceptada por uma superfície j ($q_{i\to j}$)

$$F_{ij} = \frac{q_{i\to j}}{A_i J_i}$$

- usando os conceitos definidos previamente:

$$dq_{i\to j} = I_i \cos\theta_i \, d\omega_{i\to j}$$



emissor e refletor difuso

$$J_i = \pi I_i$$

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$

$d\,\omega$ = ângulo sódido. Região do raio de uma esfera.

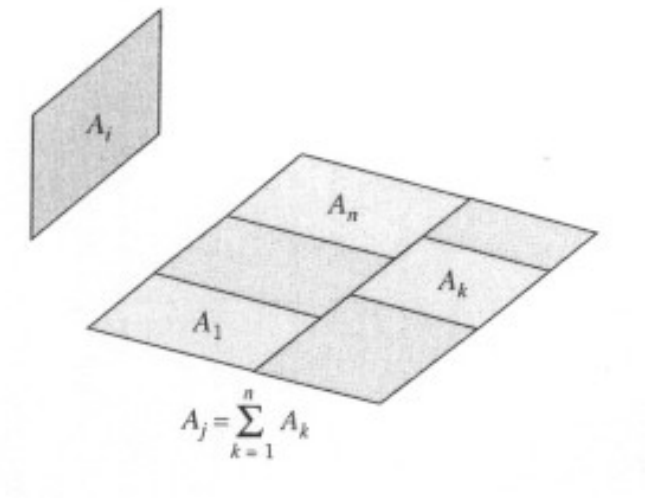
$$A_i F_{ij} = A_j F_{ji}$$

$$d\omega_{i\to j} = \frac{dA_n}{R^2} = \cos\theta_j \frac{dA_j}{R^2}$$

-Fator de forma:

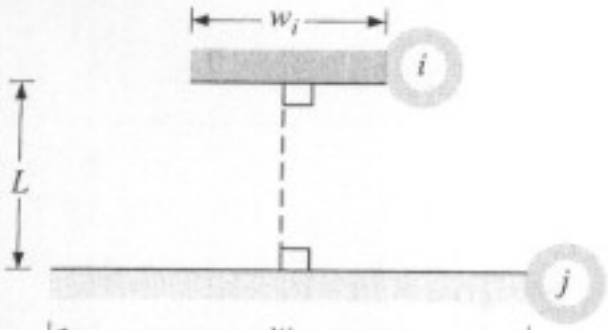
-Para radiação de uma superfície *i* para uma superfície *j*, dividida em *n* intervalos:

$$F_{i(j)} = \sum_{k=1}^{N} F_{ik}$$



- Para superfícies em cavidades fechadas: $\sum_{j=1}^{N} F_{ij} = 1$

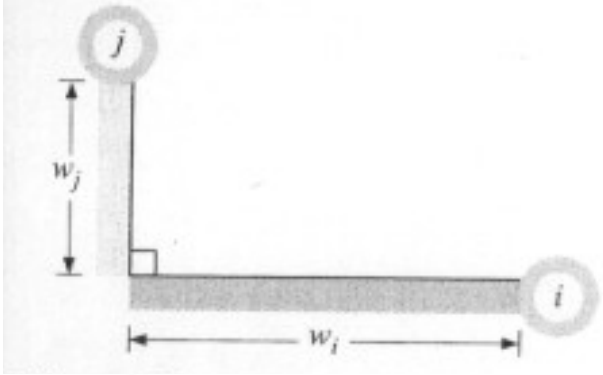
Parallel Plates with Midlines Connected by Perpendicular



$$F_{ij} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$$

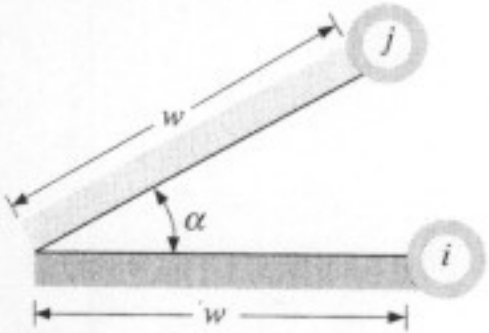
$W_i = w_i/L,\ W_j = w_j/L$

Perpendicular Plates with a Common Edge



$$F_{ij} = \frac{1 + (w_j/w_i) - [1 + (w_j/w_i)^2]^{1/2}}{2}$$

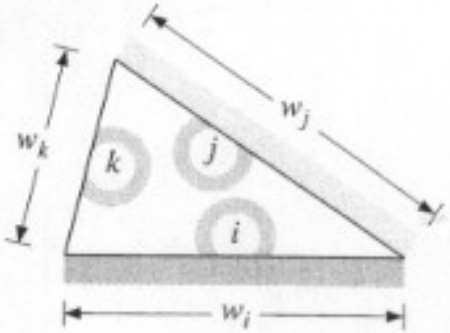
Inclined Parallel Plates of Equal Width and a Common Edge



Perpendicular Plates with a Common Edge

$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

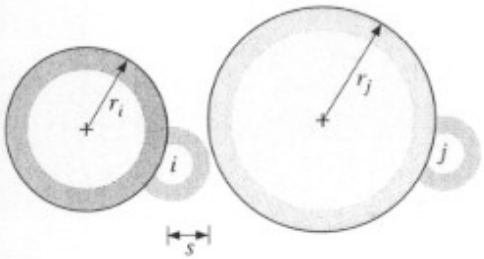
Three-Sided Enclosure



$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$

Geometry	Relation

Parallel Cylinders of Different Radii

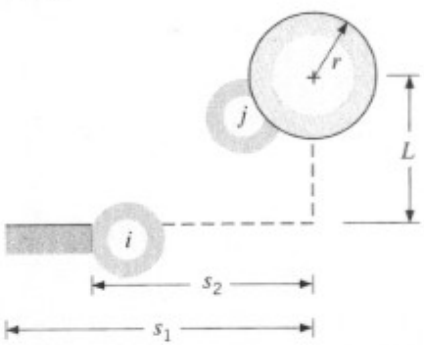


$$F_{ij} = \frac{1}{2\pi}\left\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\right\}$$

$$R = r_j/r_i,\ S = s/r_i$$

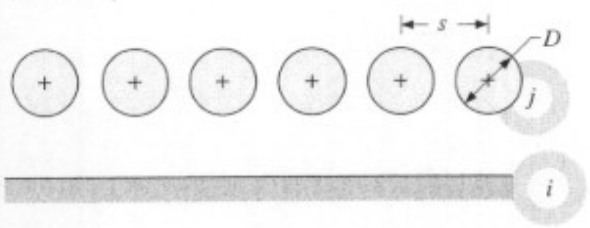
$$C = 1 + R + S$$

Cylinder and Parallel Rectangle

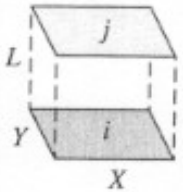
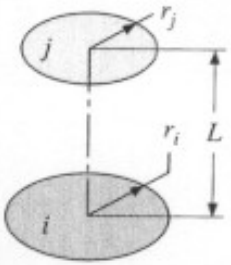
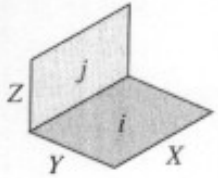


$$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$$

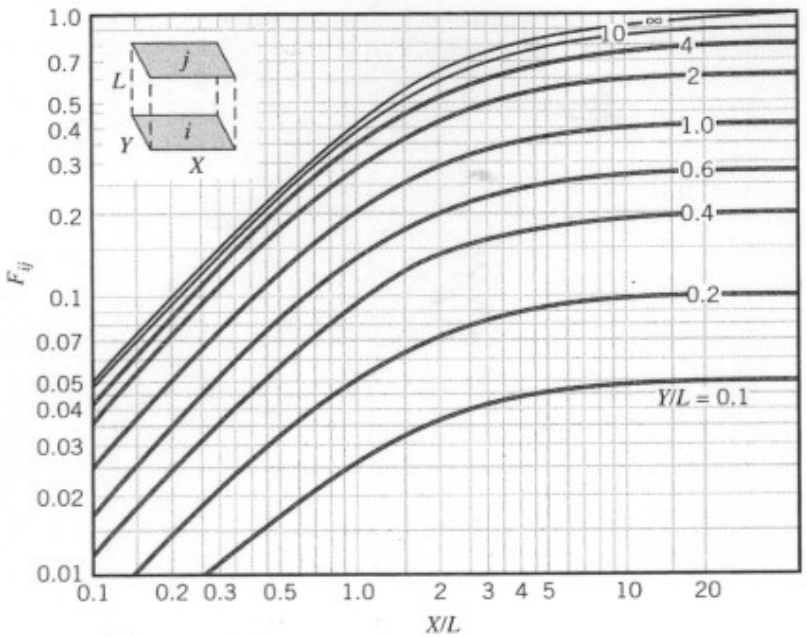
Infinite Plane and Row of Cylinders

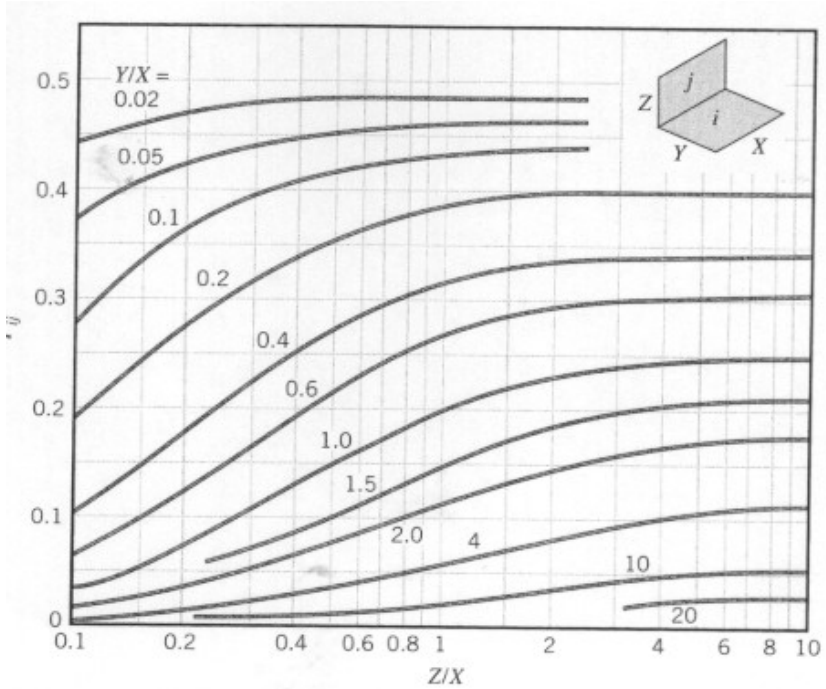


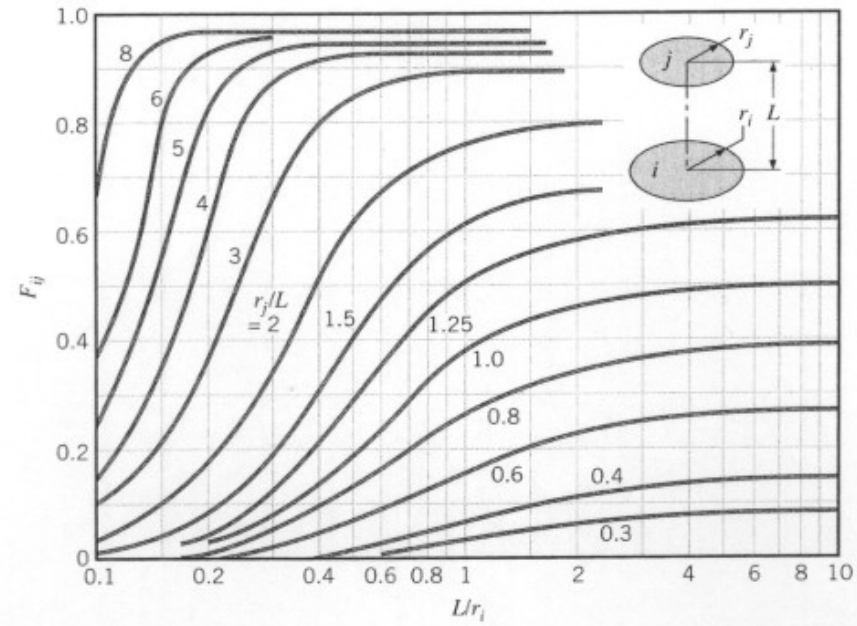
$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right)\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

Geometry	Relation
Aligned Parallel Rectangles (Figure 13.4) 	$\overline{X} = X/L,\ \overline{Y} = Y/L$ $F_{ij} = \frac{2}{\pi \overline{X}\,\overline{Y}}\left\{\ln\left[\frac{(1+\overline{X}^2)(1+\overline{Y}^2)}{1+\overline{X}^2+\overline{Y}^2}\right]^{1/2} + \overline{X}(1+\overline{Y}^2)^{1/2}\tan^{-1}\frac{\overline{X}}{(1+\overline{Y}^2)^{1/2}} + \overline{Y}(1+\overline{X}^2)^{1/2}\tan^{-1}\frac{\overline{Y}}{(1+\overline{X}^2)^{1/2}} - \overline{X}\tan^{-1}\overline{X} - \overline{Y}\tan^{-1}\overline{Y}\right\}$
Coaxial Parallel Disks (Figure 13.5) 	$R_i = r_i/L,\ R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$
Perpendicular Rectangles with a Common Edge (Figure 13.6) 	$H = Z/X,\ W = Y/X$ $F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$

Gráficos para determinação do Fator de Forma



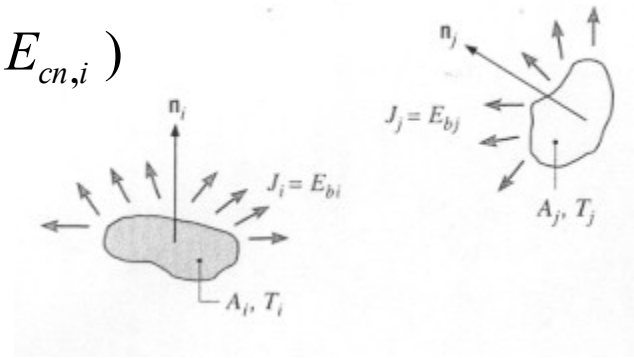




- Troca de calor por radiação entre 2 superfícies consideradas como corpo negro (CN)

$$q_{i\to j} = (A_i J_i)F_{ij} = A_i E_{cn,i} F_{ij} \qquad \text{(pois p/ CN, } J_i = E_{cn,i}\text{)}$$

analogamente $\quad q_{j\to i} = A_j E_{cn,j} F_{ji}$

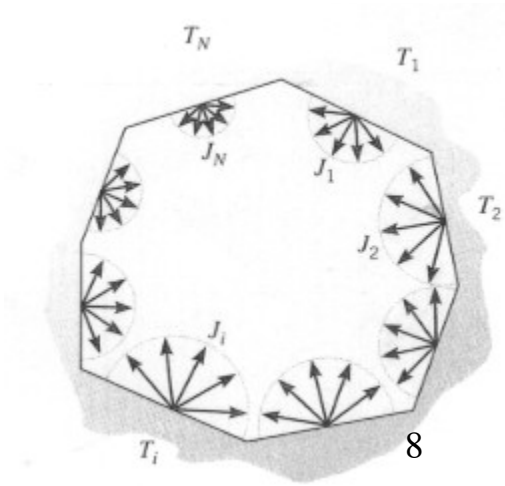


$$q_{ij} = q_{i\to j} - q_{j\to i}$$

$$q_{ij} = A_i F_{ij} \sigma (T_i^4 - T_j^4)$$

- numa cavidade fechada, composta por N superfícies de CN:

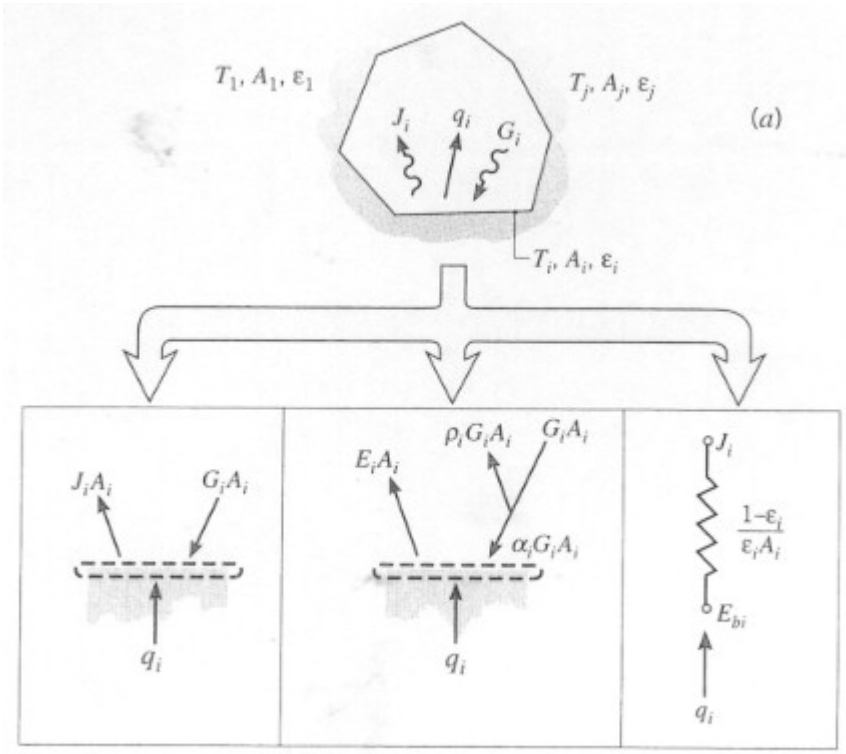
$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma (T_i^4 - T_j^4)$$



-Troca de calor entre superfícies cinzas difusas em cavidades fechadas

- neste caso existe reflexão
- hipóteses: superfícies isotérmicas, radiosidade e irradiação uniformes, superfície opaca (τ=0, $\varrho+\alpha$=1) e difusa, meio não participante, sup. cinza ($\varepsilon=\alpha$)

- fluxo líquido de radiação que deixa a superfície i: $q_i=A_i(J_i-G_i)$
- radiosidade $J_i \equiv E_i+\rho_i G_i$



Vimos que $E_i = \varepsilon_i E_{cn,i}$

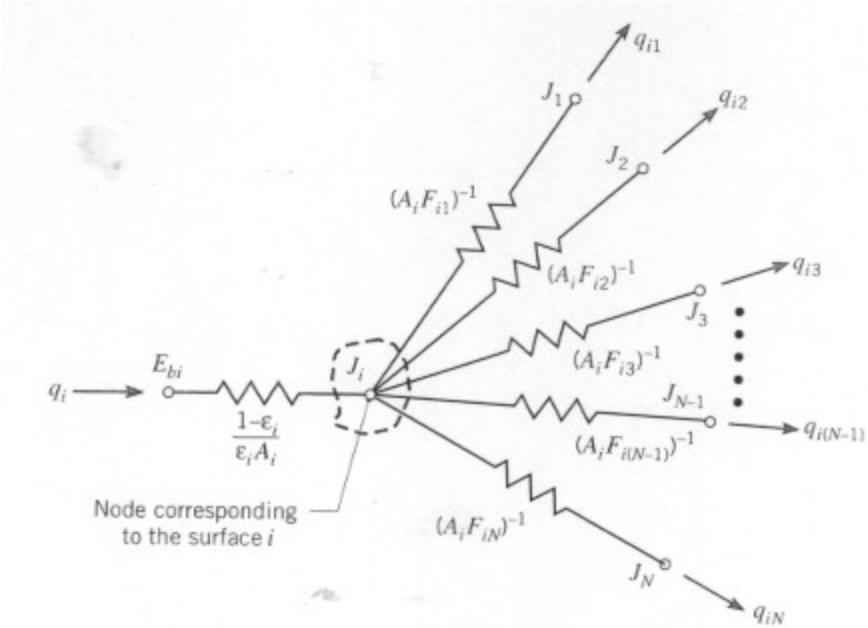
Sabendo que $\rho_i = 1-\alpha_i = 1-\varepsilon_i$

Fluxo líquido que deixa a superfície i:

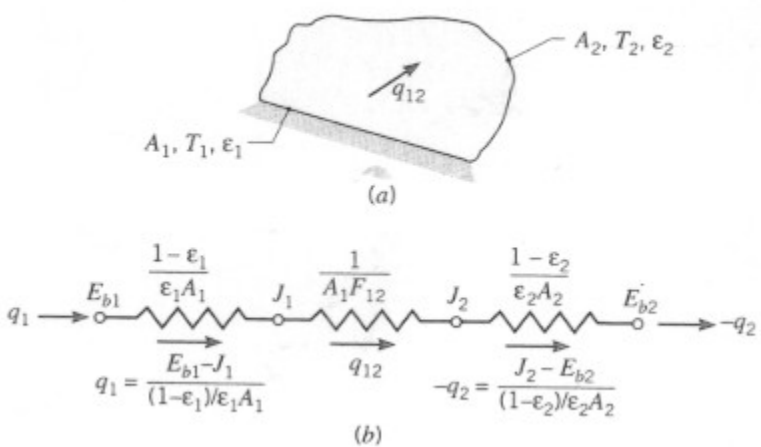
$$q_i = \frac{E_{cn,i} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i}$$

- Troca por radiação entre as superfícies da cavidade:

$$q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{(A_i F_{ij})^{-1}}$$

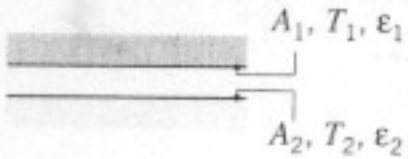


- para uma cavidade composta por 2 superfícies:



$$q_1 = -q_2 = q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

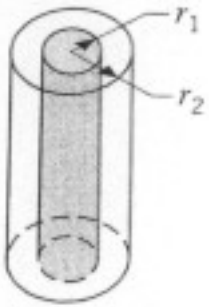
Large (Infinite) Parallel Planes



$$A_1 = A_2 = A$$
$$F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

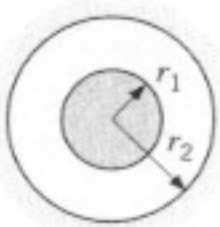
Long (Infinite) Concentric Cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$
$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)}$$

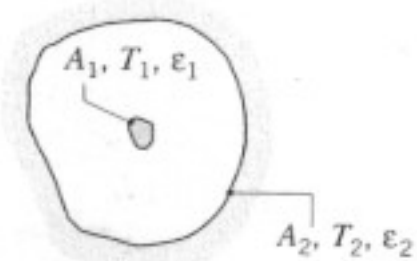
Concentric Spheres



$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$
$$F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2}$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0$$
$$F_{12} = 1$$

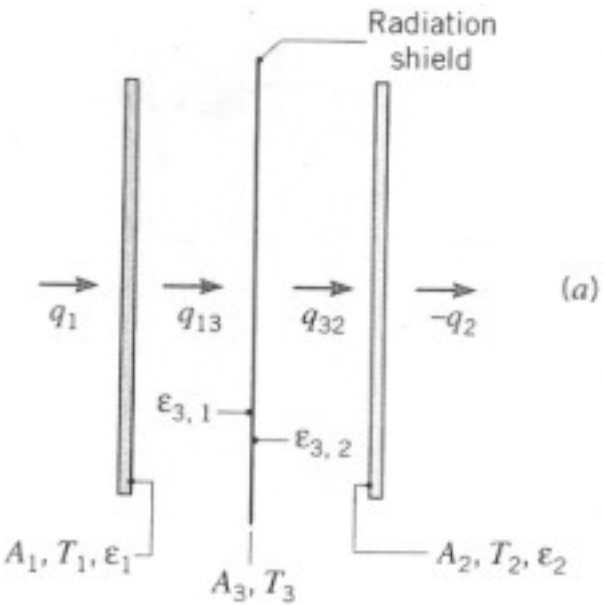
$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4)$$

-Blindagens para radiação:

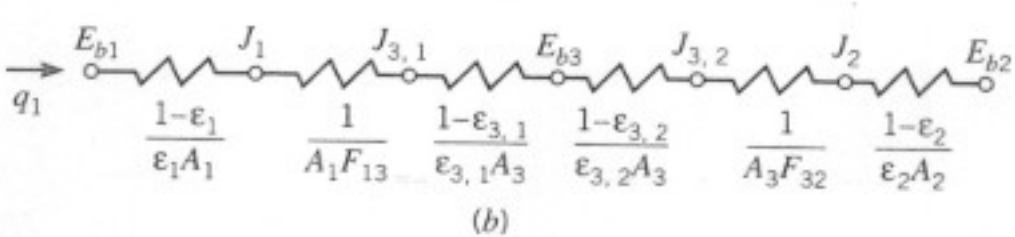
- são construídas de materiais de baixa emissividade e alta refletividade.
- usadas para reduzir a transferência líquida por radiação entre 2 superfícies

- a presença da blindagem proporciona uma resistência térmica adicional, reduzindo a troca de calor entre as superfícies

- $\varepsilon_{3,1}$ e $\varepsilon_{3,2}$ pequenos
- Sabendo que $F_{13}=F_{32}=1$



$$q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1}} + \frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2}}}$$



Observações sobre simplificações usadas:

- Superfícies idealizadas:
  - Isotérmicas
  - Cinzas
  - Emitem e refletem difusamente
  - Distribuição de radiosidade e irradiação uniformes
- Cavidades preenchidas em meios não participantes
  - Não emite, não espalha, não absorve radiação
- Permite estimativa de troca de calor por radiação com boa exatidão em muitos casos

Radiação em gases:

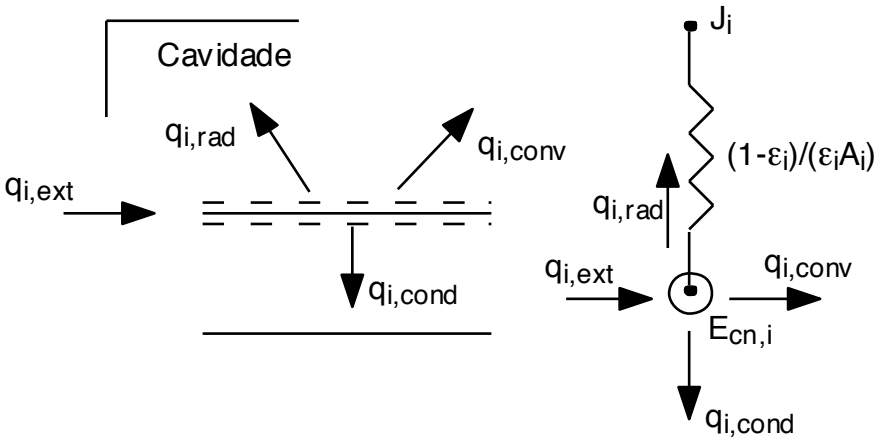
- Podem emitir e absorver
- Fenômeno volumétrico

-Troca de calor combinada:
Condução + convecção + radiação

-Em muitas aplicações práticas da engenharia, encontramos os 3 modos de transferência de calor

- O balanço de calor para o exemplo da figura é dado por:

$$q_{i,ext} = q_{i,rad} + q_{i,conv} + q_{i,cond}$$

$q_{i,rad}$ - fluxo líquido de radiação da superfície ($\propto \Delta T^4$)

$q_{i,conv}, q_{i,cond} \quad \propto \Delta T$