q_6: conduction through the cover

q_7: free convection from the cover to the room air

q_8: net radiation exchange between the outer surface of the cover and the surroundings

Comments: Design improvements are associated with (1) use of aluminized (low emissivity) surfaces for the flask and cover to reduce net radiation, and (2) evacuating the air space or using a filler material to retard free convection.

Problems

Conduction

1.1 A heat rate of 3 kW is conducted through a section of an insulating material of cross-sectional area 10 m^2 and thickness 2.5 cm. If the inner (hot) surface temperature is 415°C and the thermal conductivity of the material is 0.2 W/m · K, what is the outer surface temperature?

1.2 A concrete wall, which has a surface area of 20 m^2 and is 0.30 m thick, separates conditioned room air from ambient air. The temperature of the inner surface of the wall is maintained at 25°C, and the thermal conductivity of the concrete is 1 W/m · K.

(a) Determine the heat loss through the wall for outer surface temperatures ranging from −15°C to 38°C, which correspond to winter and summer extremes, respectively. Display your results graphically.

(b) On your graph, also plot the heat loss as a function of the outer surface temperature for wall materials having thermal conductivities of 0.75 and 1.25 W/m · K. Explain the family of curves you have obtained.

1.3 The concrete slab of a basement is 11 m long, 8 m wide, and 0.20 m thick. During the winter, temperatures are nominally 17°C and 10°C at the top and bottom surfaces, respectively. If the concrete has a thermal conductivity of 1.4 W/m · K, what is the rate of heat loss through the slab? If the basement is heated by a gas furnace operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at C_g = \$0.01 /MJ, what is the daily cost of the heat loss?

1.4 The heat flux through a wood slab 50 mm thick, whose inner and outer surface temperatures are 40 and 20°C, respectively, has been determined to be 40 W/m^2. What is the thermal conductivity of the wood?

1.5 The inner and outer surface temperatures of a glass window 5 mm thick are 15 and 5°C. What is the heat loss through a window that is 1 m by 3 m on a side? The thermal conductivity of glass is 1.4 W/m · K.

1.6 A glass window of width W = 1 m and height H = 2 m is 5 mm thick and has a thermal conductivity of k_g = 1.4 W/m · K. If the inner and outer surface temperatures of the glass are 15°C and −20°C, respectively, on a cold winter day, what is the rate of heat loss through the glass? To reduce heat loss through windows, it is customary to use a double pane construction in which adjoining panes are separated by an air space. If the spacing is 10 mm and the glass surfaces in contact with the air have temperatures of 10°C and −15°C, what is the rate of heat loss from a 1 m × 2 m window? The thermal conductivity of air is k_a = 0.024 W/m · K.

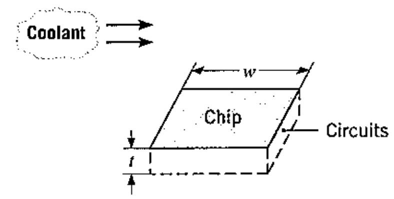
1.7 A freezer compartment consists of a cubical cavity that is 2 m on a side. Assume the bottom to be perfectly insulated. What is the minimum thickness of styrofoam insulation (k = 0.030 W/m · K) that must be applied to the top and side walls to ensure a heat load of less than 500 W, when the inner and outer surfaces are −10 and 35°C?

1.8 An inexpensive food and beverage container is fabricated from 25-mm-thick polystyrene (k = 0.023 W/m·K) and has interior dimensions of 0.8 m × 0.6 m × 0.6 m. Under conditions for which an inner surface temperature of approximately 2°C is maintained by an ice–water mixture and an outer surface temperature of 20°C is maintained by the ambient, what is the heat flux through the container wall? Assuming negligible heat gain through the 0.8 m × 0.6 m base of the cooler, what is the total heat load for the prescribed conditions?

1.9 What is the thickness required of a masonry wall having thermal conductivity 0.75 W/m · K if the heat rate is to be 80% of the heat rate through a composite structural wall having a thermal conductivity of 0.25 W/m · K and a thickness of 100 mm? Both walls are subjected to the same surface temperature difference.

1.10 The 5-mm-thick bottom of a 200-mm-diameter pan may be made from aluminum (k = 240 W/m·K) or copper (k = 390 W/m·K). When used to boil water, the surface of the bottom exposed to the water is nominally at 110°C. If heat is transferred from the stove to the pan at a rate of 600 W, what is the temperature of the surface in contact with the stove for each of the two materials?

1.11 A square silicon chip (k = 150 W/m · K) is of width w = 5 mm on a side and of thickness t = 1 mm. The chip is mounted in a substrate such that its side and back surfaces are insulated, while the front surface is exposed to a coolant.

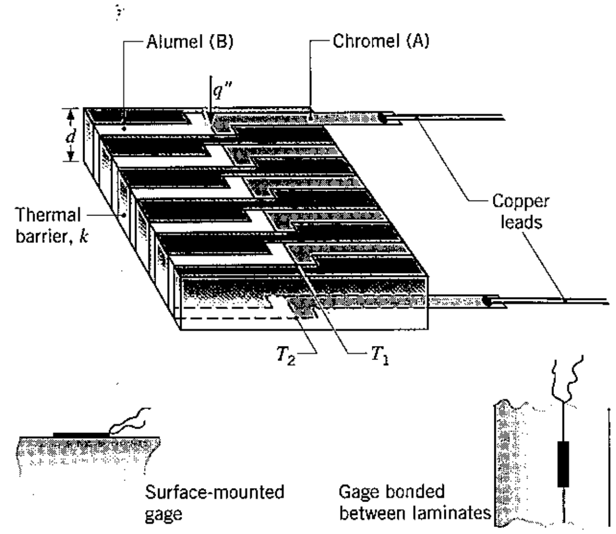


If 4 W are being dissipated in circuits mounted to the back surface of the chip, what is the steady-state temperature difference between back and front surfaces?

1.12 A gage for measuring heat flux to a surface or through a laminated material employs thin-film, chromel/alumel (type K) thermocouples deposited on the upper and lower surfaces of a wafer with a thermal conductivity of 1.4 W/m · K and a thickness of 0.25 mm.

(a) Determine the heat flux q'' through the gage when the voltage output at the copper leads is 350 μV. The Seebeck coefficient of the type-K thermocouple materials is approximately 40 μV/°C.

(b) What precaution should you take in using a gage of this nature to measure heat flow through the laminated structure shown above?



Convection

1.13 You've experienced convection cooling if you've ever extended your hand out the window of a moving vehicle or into a flowing water stream. With the surface of your hand at a temperature of 30°C, determine the convection heat flux for (a) a vehicle speed of 35 km/h in air at −5°C with a convection coefficient of 40 W/m^2 · K and (b) a velocity of 0.2 m/s in a water stream at 10°C with a convection coefficient of 900 W/m^2 · K. Which condition would *feel* colder? Contrast these results with a heat loss of approximately 30 W/m^2 under normal room conditions.

1.14 Air at 40°C flows over a long, 25-mm diameter cylinder with an embedded electrical heater. In a series of tests, measurements were made of the power per unit length, P', required to maintain the cylinder surface temperature at 300°C for different freestream velocities V of the air. The results are as follows:

Air velocity, V (m/s)	1	2	4	8	12
Power, P' (W/m)	450	658	983	1507	1963

(a) Determine the convection coefficient for each velocity, and display your results graphically.

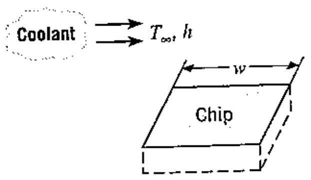
(b) Assuming the dependence of the convection coefficient on the velocity to be of the form $h = CV^n$, determine the parameters C and n from the results of part (a).

1.15 An electric resistance heater is embedded in a long cylinder of diameter 30 mm. When water with a temperature of 25°C and velocity of 1 m/s flows crosswise over the cylinder, the power per unit length required to maintain the surface at a uniform temperature of 90°C is 28 kW/m. When air, also at 25°C, but with a velocity of 10 m/s is flowing, the power per unit length required to maintain the same surface temperature is 400 W/m. Calculate and compare the convection coefficients for the flows of water and air.

1.16 A cartridge electrical heater is shaped as a cylinder of length L = 200 mm and outer diameter D = 20 mm. Under normal operating conditions the heater dissipates 2 kW, while submerged in a water flow that is at 20°C and provides a convection heat transfer coefficient of h = 5000 W/m$^2 \cdot$ K. Neglecting heat transfer from the ends of the heater, determine its surface temperature T_s. If the water flow is inadvertently terminated while the heater continues to operate, the heater surface is exposed to air that is also at 20°C but for which h = 50 W/m$^2 \cdot$ K. What is the corresponding surface temperature? What are the consequences of such an event?

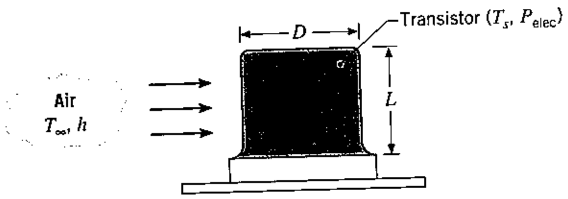
1.17 A common procedure for measuring the velocity of an air stream involves insertion of an electrically heated wire (called a *hot-wire anemometer*) into the air flow, with the axis of the wire oriented perpendicular to the flow direction. The electrical energy dissipated in the wire is assumed to be transferred to the air by forced convection. Hence, for a prescribed electrical power, the temperature of the wire depends on the convection coefficient, which, in turn, depends on the velocity of the air. Consider a wire of length L = 20 mm and diameter D = 0.5 mm, for which a calibration of the form, $V = 6.25 \times 10^{-5} h^2$, has been determined. The velocity V and the convection coefficient h have units of m/s and W/m$^2 \cdot$ K, respectively. In an application involving air at a temperature of T_∞ = 25°C, the surface temperature of the anemometer is maintained at T_s = 75°C with a voltage drop of 5 V and an electric current of 0.1 A. What is the velocity of the air?

1.18 A square isothermal chip is of width w = 5 mm on a side and is mounted in a substrate such that its side and back surfaces are well insulated, while the front surface is exposed to the flow of a coolant at T_∞ = 15°C. From reliability considerations, the chip temperature must not exceed T = 85°C.



If the coolant is air and the corresponding convection coefficient is h = 200 W/m$^2 \cdot$ K, what is the maximum allowable chip power? If the coolant is a dielectric liquid for which h = 3000 W/m$^2 \cdot$ K, what is the maximum allowable power?

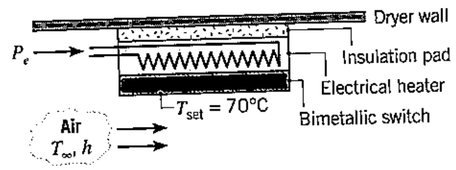
1.19 The case of a power transistor, which is of length L = 10 mm and diameter D = 12 mm, is cooled by an air stream of temperature T_∞ = 25°C.



Under conditions for which the air maintains an average convection coefficient of h = 100 W/m$^2 \cdot$ K on the surface of the case, what is the maximum allowable power dissipation if the surface temperature is not to exceed 85°C?

1.20 The use of impinging air jets is proposed as a means of effectively cooling high-power logic chips in a computer. However, before the technique can be implemented, the convection coefficient associated with jet impingement on a chip surface must be known. Design an experiment that could be used to determine convection coefficients associated with air jet impingement on a chip measuring approximately 10 mm by 10 mm on a side.

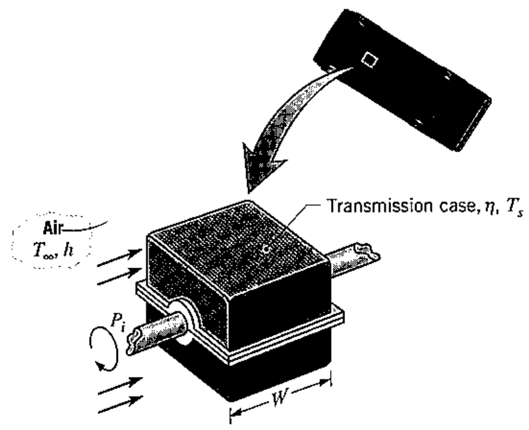
1.21 The temperature controller for a clothes dryer consists of a bimetallic switch mounted on an electrical heater attached to a wall-mounted insulation pad.



The switch is set to open at 70°C, the maximum dryer air temperature. In order to operate the dryer at a lower air temperature, sufficient power is supplied to the heater such that the switch reaches 70°C (T_{set}) when the air temperature T_∞ is less than T_{set}. If the convection heat transfer coefficient between the air and the exposed switch surface of 30 mm^2 is 25 W/m$^2 \cdot$ K, how much heater power P_e is required when the desired dryer air temperature is T_∞ = 50°C?

1.22 The free convection heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assuming the plate is isothermal and radiation exchange with its surroundings is negligible, evaluate the convection coefficient at the instant of time when the plate temperature is 225°C and the change in plate temperature with time (dT/dt) is − 0.022 K/s. The ambient air temperature is 25°C and the plate measures 0.3 × 0.3 m with a mass of 3.75 kg and a specific heat of 2770 J/kg · K.

1.23 A transmission case measures W = 0.30 m on a side and receives a power input of P_i = 150 hp from the engine.



If the transmission efficiency is η = 0.93 and air flow over the case corresponds to T_∞ = 30°C and h = 200 W/m$^2 \cdot$ K, what is the surface temperature of the transmission?

Radiation

1.24 Under conditions for which the same room temperature is maintained by a heating or cooling system, it is not uncommon for a person to feel chilled in the winter but comfortable in the summer. Provide a plausible explanation for this situation (with supporting calculations) by considering a room whose air temperature is maintained at 20°C throughout the year, while the walls of the room are nominally at 27°C and 14°C in the summer and winter, respectively. The exposed surface of a person in the room may be assumed to be at a temperature of 32°C throughout the year and to have an emissivity of 0.90. The coefficient associated with heat transfer by natural convection between the person and the room air is approximately 2 W/m$^2 \cdot$ K.

1.25 A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150 W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?

1.26 An instrumentation package has a spherical outer surface of diameter D = 100 mm and emissivity ε = 0.25. The package is placed in a large space simulation chamber whose walls are maintained at 77 K. If operation of the electronic components is restricted to the temperature range $40 \le T \le 85$°C, what is the range of acceptable power dissipation for the package? Display your results graphically, showing also the effect of variations in the emissivity by considering values of 0.20 and 0.30.

1.27 A surface of area 0.5 m^2, emissivity 0.8, and temperature 150°C is placed in a large, evacuated chamber whose walls are maintained at 25°C. What is the rate at which radiation is emitted by the surface? What is the net rate at which radiation is exchanged between the surface and the chamber walls?

1.28 An overhead 25-m-long, uninsulated industrial steam pipe of 100 mm diameter is routed through a building whose walls and air are at 25°C. Pressurized steam maintains a pipe surface temperature of 150°C, and the coefficient associated with natural convection is h = 10 W/m$^2 \cdot$ K. The surface emissivity is ε = 0.8.

(a) What is the rate of heat loss from the steam line?

(b) If the steam is generated in a gas-fired boiler operating at an efficiency of η_f = 0.90 and natural gas is priced at C_g = \$0.01 per MJ, what is the annual cost of heat loss from the line?

1.29 If $T_s \approx T_{sur}$ in Equation 1.9, the radiation heat transfer coefficient may be approximated as

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3$$

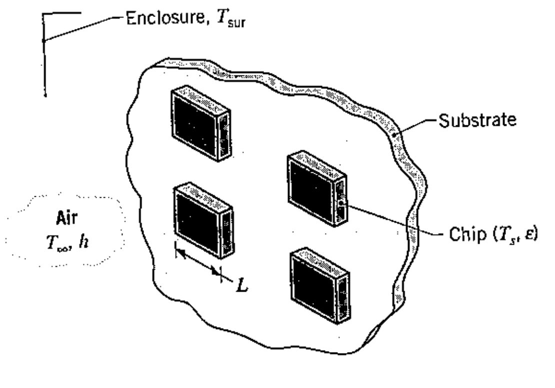
where $\bar{T} \equiv (T_s + T_{sur})/2$. We wish to assess the validity of this approximation by comparing values of h_r and $h_{r,a}$ for the following conditions. In each case represent your results graphically and comment on the validity of the approximation.

(a) Consider a surface of either polished aluminum (ε = 0.05) or black paint (ε = 0.9), whose temperature may exceed that of the surroundings (T_{sur} = 25°C) by 10 to 100°C. Also compare your results with values of the coefficient associated with free convection in air ($T_\infty = T_{sur}$), where h (W/m$^2 \cdot$K) = 0.98 $\Delta T^{1/3}$.

(b) Consider initial conditions associated with placing a workpiece at $T_s = 25°C$ in a large furnace whose wall temperature may be varied over the range $100 \le T_{sur} \le 1000°C$. According to the surface finish or coating, its emissivity may assume values of 0.05, 0.2, and 0.9. For each emissivity, plot the relative error, $(h_r - h_{r,a})/h_r$, as a function of the furnace temperature.

30 Consider the conditions of Problem 1.18. With heat transfer by convection to air, the maximum allowable chip power is found to be 0.35 W. If consideration is also given to net heat transfer by radiation from the chip surface to large surroundings at 15°C, what is the percentage increase in the maximum allowable chip power afforded by this consideration? The chip surface has an emissivity of 0.9.

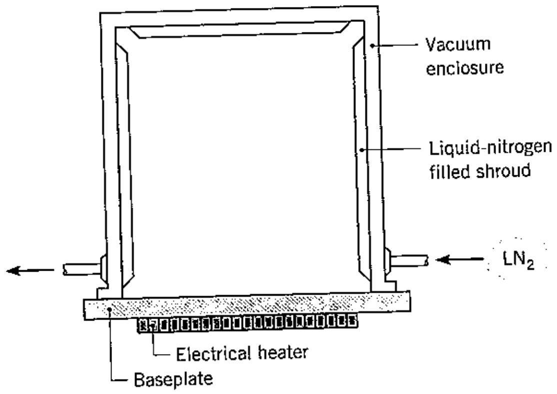
.31 Chips of width $L = 15$ mm on a side are mounted to a substrate that is installed in an enclosure whose walls and air are maintained at a temperature of $T_{sur} = T_\infty = 25°C$. The chips have an emissivity of $\varepsilon = 0.60$ and a maximum allowable temperature of $T_s = 85°C$.



(a) If heat is rejected from the chips by radiation and natural convection, what is the maximum operating power of each chip? The convection coefficient depends on the chip-to-air temperature difference and may be approximated as $h = C(T_s - T_\infty)^{1/4}$, where $C = 4.2$ W/m$^2 \cdot$ K$^{5/4}$.

(b) If a fan is used to maintain air flow through the enclosure and heat transfer is by forced convection, with $h = 250$ W/m$^2 \cdot$ K, what is the maximum operating power?

1.32 A vacuum system, as used in sputtering electrically conducting thin films on microcircuits, is comprised of a baseplate maintained by an electrical heater at 300 K and a shroud within the enclosure maintained at 77 K by a liquid-nitrogen coolant loop. The baseplate, insulated on the lower side, is 0.3 m in diameter and has an emissivity of 0.25.



(a) How much electrical power must be provided to the baseplate heater?

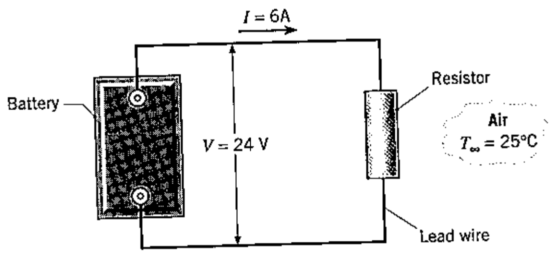
(b) At what rate must liquid nitrogen be supplied to the shroud if its heat of vaporization is 125 kJ/kg?

(c) To reduce the liquid-nitrogen consumption, it is proposed to bond a thin sheet of aluminum foil ($\varepsilon = 0.09$) to the baseplate. Will this have the desired effect?

1.33 Consider the transmission case of Problem 1.23, but now allow for radiation exchange with the ground/chassis, which may be approximated as large surroundings at $T_{sur} = 30°C$. If the emissivity of the case is $\varepsilon = 0.80$, what is the surface temperature?

Energy Balance and Multimode Effects

1.34 An electrical resistor is connected to a battery, as shown schematically. After a brief transient, the resistor assumes a nearly uniform, steady-state temperature of 95°C, while the battery and lead wires remain at the ambient temperature of 25°C. Neglect the electrical resistance of the lead wires.



(a) Consider the resistor as a system about which a control surface is placed and Equation 1.11a is applied. Determine the corresponding values of $\dot{E}_{in}$ (W), $\dot{E}_g$ (W), $\dot{E}_{out}$ (W), and $\dot{E}_{st}$ (W). If a control surface is placed about the entire system, what are the values of $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$?

(b) If electrical energy is dissipated uniformly within the resistor, which is a cylinder of diameter $D = 60$ mm and length $L = 250$ mm, what is the volumetric heat generation rate, $\dot{q}$ (W/m^3)?

(c) Neglecting radiation from the resistor, what is the convection coefficient?

1.35 An aluminum plate 4 mm thick is mounted in a horizontal position, and its bottom surface is well insulated. A special, thin coating is applied to the top surface such that it absorbs 80% of any incident solar radiation, while having an emissivity of 0.25. The density ρ and specific heat c of aluminum are known to be 2700 kg/m^3 and 900 J/kg$\cdot$K, respectively.

(a) Consider conditions for which the plate is at a temperature of 25°C and its top surface is suddenly exposed to ambient air at $T_\infty = 20°C$ and to solar radiation that provides an incident flux of 900 W/m^2. The convection heat transfer coefficient between the surface and the air is $h = 20$ W/m$^2\cdot$K. What is the initial rate of change of the plate temperature?

(b) What will be the equilibrium temperature of the plate when steady-state conditions are reached?

$\boxed{\text{(c)}}$ The surface radiative properties depend on the specific nature of the applied coating. Compute and plot the steady-state temperature as a function of the emissivity for $0.05 \le \varepsilon \le 1$, with all other conditions remaining as prescribed. Repeat your calculations for values of $\alpha_S = 0.5$ and 1.0, and plot the results with those obtained for $\alpha_S = 0.8$. If the intent is to maximize the plate temperature, what is the most desirable combination of the plate emissivity and its absorptivity to solar radiation?

1.36 In an orbiting space station, an electronic package is housed in a compartment having a surface area, $A_s = 1$ m^2, which is exposed to space. Under normal operating conditions, the electronics dissipate 1 kW, all of which must be transferred from the exposed surface to space. If the surface emissivity is 1.0 and the surface is not exposed to the sun, what is its steady-state temperature? If the surface is exposed to a solar flux of 750 W/m^2 and its absorptivity to solar radiation is 0.25, what is its steady-state temperature?

1.37 The energy consumption associated with a home water heater has two components: (i) the energy that must be supplied to bring the temperature of groundwater to the heater storage temperature, as it is introduced to replace hot water that has been used, and (ii) the energy needed to compensate for heat losses incurred while the water is stored at the prescribed temperature. In this problem, we will evaluate the first of these components for a family of four, whose daily hot water consumption is approximately 100 gallons. If groundwater is available at 15°C, what is the annual energy consumption associated with heating the water to a storage temperature of 55°C? For a unit electrical power cost of \$0.08/kWh, what is the annual cost associated with supplying hot water by means of (a) electric resistance heating or (b) a heat pump having a COP of 3.

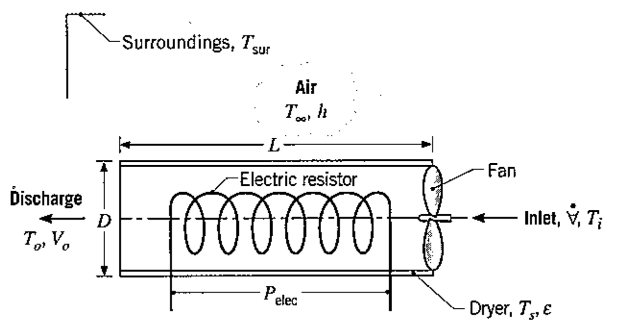
1.38 Three electric resistance heaters of length $L = 250$ mm and diameter $D = 25$ mm are submerged in a 10 gallon tank of water, which is initially at 295 K. The water may be assumed to have a density and specific heat of $\rho = 990$ kg/m^3 and $c = 4180$ J/kg$\cdot$K.

(a) If the heaters are activated, each dissipating $q_1 = 500$ W, estimate the time required to bring the water to a temperature of 335 K.

(b) If the natural convection coefficient is given by an expression of the form $h = 370(T_s - T)^{1/3}$, where T_s and T are temperatures of the heater surface and water, respectively, what is the temperature of each heater shortly after activation and just before deactivation? Units of h and $(T_s - T)$ are W/m$^2\cdot$K and K, respectively.

(c) If the heaters are inadvertently activated when the tank is empty, the natural convection coefficient associated with heat transfer to the ambient air at $T_\infty = 300$ K may be approximated as $h = 0.70(T_s - T_\infty)^{1/3}$. If the temperature of the tank walls is also 300 K and the emissivity of the heater surface is $\varepsilon = 0.85$, what is the surface temperature of each heater under steady-state conditions?

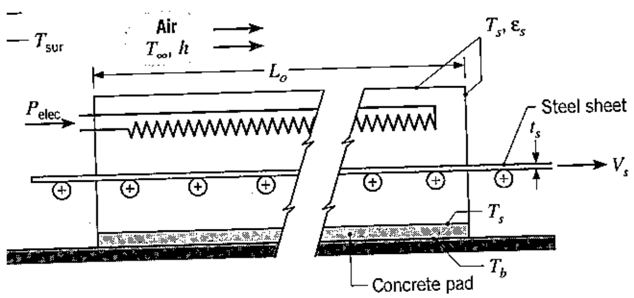
1.39 A hair dryer may be idealized as a circular duct through which a small fan draws ambient air and within which the air is heated as it flows over a coiled electric resistance wire.



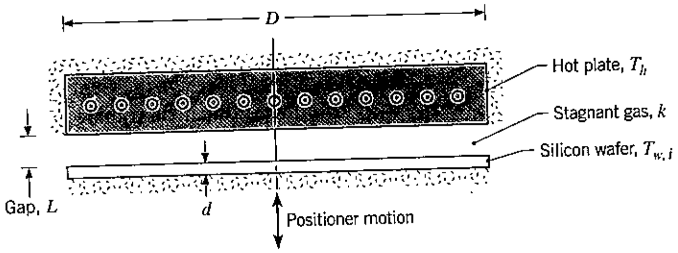
(a) If a dryer is designed to operate with an electric power consumption of $P_{elec} = 500$ W and to heat air from an ambient temperature of $T_i = 20°C$ to a discharge temperature of $T_o = 45°C$, at what volumetric flow rate $\forall$ should the fan operate? Heat loss from the casing to the ambient air and the surroundings may be neglected. If the duct has a diameter of $D = 70$ mm, what is the discharge velocity V_o of the air? The density and specific heat of the air may be approximated as $\rho = 1.10\ \mathrm{kg/m^3}$ and $c_p = 1007\ \mathrm{J/kg\cdot K}$, respectively.

(b) Consider a dryer duct length of $L = 150$ mm and a surface emissivity of $\varepsilon = 0.8$. If the coefficient associated with heat transfer by natural convection from the casing to the ambient air is $h = 4\ \mathrm{W/m^2\cdot K}$ and the temperature of the air and the surroundings is $T_\infty = T_{sur} = 20°C$, confirm that the heat loss from the casing is, in fact, negligible. The casing may be assumed to have an average surface temperature of $T_s = 40°C$.

1.40 In one stage of an annealing process, 304 stainless steel sheet is taken from 300 K to 1250 K as it passes through an electrically heated oven at a speed of $V_s = 10$ mm/s. The sheet thickness and width are $t_s = 8$ mm and $W_s = 2$ m, respectively, while the height, width, and length of the oven are $H_o = 2$ m, $W_o = 2.4$ m, and $L_o = 25$ m, respectively. The top and four sides of the oven are exposed to ambient air and large surroundings, each at 300 K, and the corresponding surface temperature, convection coefficient, and emissivity are $T_s = 350$ K, $h = 10\ \mathrm{W/m^2\cdot K}$ and $\varepsilon_s = 0.8$. The bottom surface of the oven is also at 350 K and rests on a 0.5-m thick concrete pad whose base is at 300 K. Estimate the required electric power input, P_{elec}, to the oven.



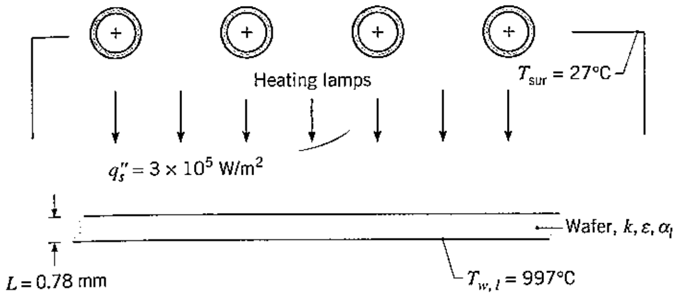
1.41 Annealing, an important step in semiconductor materials processing, can be accomplished by rapidly heating the silicon wafer to a high temperature for a short period of time. The schematic shows a method involving use of a hot plate operating at an elevated temperature T_h. The wafer, initially at a temperature of $T_{w,i}$, is suddenly positioned at a gap separation distance L from the hot plate. The purpose of the analysis is to compare the heat fluxes by conduction through the gas within the gap and by radiation exchange between the hot plate and the cool wafer. The initial time rate of change in the temperature of the wafer, $(dT_w/dt)_i$, is also of interest. Approximating the surfaces of the hot plate and the wafer as blackbodies and assuming their diameter D to be much larger than the spacing L, the radiative heat flux may be expressed as $q''_{rad} = \sigma(T_h^4 - T_w^4)$. The silicon wafer has a thickness of $d = 0.78$ mm, a density of $2700\ \mathrm{kg/m^3}$, and a specific heat of $875\ \mathrm{J/kg\cdot K}$. The thermal conductivity of the gas in the gap is $0.0436\ \mathrm{W/m\cdot K}$.



(a) For $T_h = 600°C$ and $T_{w,i} = 20°C$, calculate the radiative heat flux and the heat flux by conduction across a gap distance of $L = 0.2$ mm. Also determine the value of $(dT_w/dt)_i$ resulting from each of the heating modes.

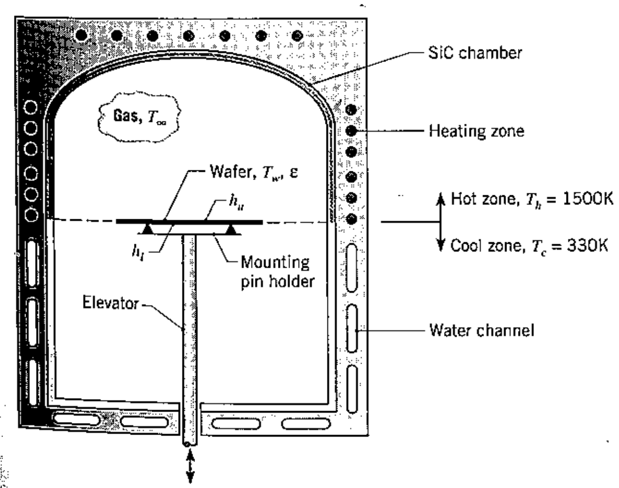
(b) For gap distances of 0.2, 0.5, and 1.0 mm, determine the heat fluxes and temperature-time change as a function of the hot plate temperature for $300 \le T_h \le 1300°C$. Display your results graphically. Comment on the relative importance of the two heat transfer modes and the effect of the gap distance on the heating process. Under what conditions could a wafer be heated to 900°C in less than 10 seconds?

1.42 In the thermal processing of semiconductor materials, annealing is accomplished by heating a silicon wafer according to a temperature-time recipe and then maintaining a fixed elevated temperature for a prescribed period of time. For the process tool arrangement shown as follows, the wafer is in an evacuated chamber whose walls are maintained at 27°C and within which heating lamps maintain a radiant flux q''_s at its upper surface. The wafer is 0.78 mm thick, has a thermal conductivity of $30\ \mathrm{W/m\cdot K}$, and an emissivity that equals its absorptivity to the radiant flux ($\varepsilon = \alpha_l = 0.65$). For $q''_s = 3.0 \times 10^5\ \mathrm{W/m^2}$, the temperature on its lower surface is measured by a radiation thermometer and found to have a value of $T_{w,l} = 997°C$.



To avoid warping the wafer and inducing slip planes in the crystal structure, the temperature difference across the thickness of the wafer must be less than 2°C. Is this condition being met?

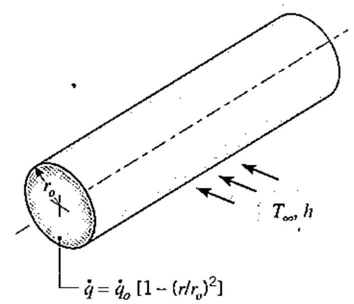
1.43 A furnace for processing semiconductor materials is formed by a silicon carbide chamber that is zone heated on the top section and cooled on the lower section. With the elevator in the lowest position, a robot arm inserts the silicon wafer on the mounting pins. In a production operation, the wafer is rapidly moved toward the hot zone to achieve the temperature-time history required for the process recipe. In this position the top and bottom surfaces of the wafer exchange radiation with the hot and cool zones, respectively, of the chamber. The zone temperatures are $T_h = 1500$ K and $T_c = 330$ K, and the emissivity and thickness of the wafer are $\varepsilon = 0.65$ and $d = 0.78$ mm, respectively. With the ambient gas at $T_\infty = 700$ K, convection coefficients at the upper and lower surfaces of the wafer are 8 and $4\ \mathrm{W/m^2\cdot K}$, respectively. The silicon wafer has a density of $2700\ \mathrm{kg/m^3}$ and a specific heat of $875\ \mathrm{J/kg\cdot K}$.



(a) For an initial condition corresponding to a wafer temperature of $T_{w,i} = 300$ K and the position of the wafer shown schematically, determine the corresponding time rate of change of the wafer temperature, $(dT_w/dt)_i$.

(b) Determine the steady-state temperature reached by the wafer if it remains in this position. How significant is convection heat transfer for this situation? Sketch how you would expect the wafer temperature to vary as a function of vertical distance.

1.44 Radioactive wastes are packed in a long, thin-walled cylindrical container. The wastes generate thermal energy nonuniformly according to the relation $\dot{q} = \dot{q}_o[1 - (r/r_o)^2]$, where $\dot{q}$ is the local rate of energy generation per unit volume, $\dot{q}_o$ is a constant, and r_o is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at T_∞ and provides a uniform convection coefficient h.



Obtain an expression for the total rate at which energy is generated in a unit length of the container. Use this result to obtain an expression for the temperature T_s of the container wall.

1.45 Consider the conducting rod of Example 1.3 under steady-state conditions. As suggested in Comment 3, the temperature of the rod may be controlled by varying the speed of air flow over the rod, which, in turn, alters the convection heat transfer coefficient. To consider the effect of the convection coefficient, generate plots of T versus I for values of $h = 50$, 100, and $250\ \mathrm{W/m^2\cdot K}$. Would variations in the surface emissivity have a significant effect on the rod temperature?

1.46 A long bus bar of diameter D is installed in a large conduit having a surface temperature of 30°C and in which the ambient air temperature is $T_\infty = 30°C$. The electrical resistivity, $\rho_e(\mu\Omega\cdot m)$, of the bar material is a function of temperature, $\rho_e = \rho_{e,o}[1 + \alpha(T - T_o)]$, where $\rho_{e,o} = 0.0171\ \mu\Omega\cdot m$, $T_o = 25°C$, and $\alpha =$

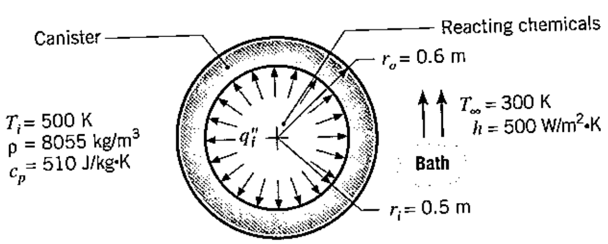
0.00396 K^{-1}. The bar experiences free convection in the ambient air, and the convection coefficient depends on the bar diameter, as well as on the difference between the surface and ambient temperatures. The governing relation is of the form, $h = CD^{-0.25}(T - T_\infty)^{0.25}$, where $C = 1.21\ W \cdot m^{-1.75} \cdot K^{-1.25}$. The emissivity of the bar surface is $\varepsilon = 0.85$.

(a) Recognizing that the electrical resistance per unit length of the bar is $R'_e = \rho_e/A_c$, where A_c is its cross-sectional area, calculate the current-carrying capacity of a 20-mm diameter bus bar if its temperature is not to exceed 65°C. Compare the relative importance of heat transfer by free convection and radiation exchange.

(b) To assess the trade-off between current-carrying capacity, operating temperature, and bar diameter, for diameters of 10, 20, and 40 mm, plot the bar temperature T as a function of current for the range $100 \le I \le 5000$ A. Also plot the ratio of the heat transfer by convection to the total heat transfer.

1.47 A small sphere of reference-grade iron with a specific heat of 447 J/kg · K and a mass of 0.515 kg is suddenly immersed in a water-ice mixture. Fine thermocouple wires suspend the sphere, and the temperature is observed to change from 15 to 14°C in 6.35 s. The experiment is repeated with a metallic sphere of the same diameter, but of unknown composition with a mass of 1.263 kg. If the same observed temperature change occurs in 4.59 s, what is the specific heat of the unknown material?

1.48 A spherical, stainless steel (AISI 302) canister is used to store reacting chemicals that provide for a uniform heat flux q''_i to its inner surface. The canister is suddenly submerged in a liquid bath of temperature $T_\infty < T_i$, where T_i is the initial temperature of the canister wall.



(a) Assuming negligible temperature gradients in the canister wall and a constant heat flux q''_i, develop an equation that governs the variation of the wall temperature with time during the transient process. What is the initial rate of change of the wall temperature if $q''_i = 10^5\ W/m^2$?

(b) What is the steady-state temperature of the wall?

(c) The convection coefficient depends on the velocity associated with fluid flow over the canister and whether or not the wall temperature is large enough to induce boiling in the liquid. Compute and plot the steady-state temperature as a function of h for the range $100 \le h \le 10{,}000\ W/m^2 \cdot K$. Is there a value of h below which operation would be unacceptable?

1.49 Liquid oxygen, which has a boiling point of 90 K and a latent heat of vaporization of 214 kJ/kg, is stored in a spherical container whose outer surface is of 500-mm diameter and at a temperature of −10°C. The container is housed in a laboratory whose air and walls are at 25°C.

(a) If the surface emissivity is 0.20 and the heat transfer coefficient associated with free convection at the outer surface of the container is 10 W/m^2 · K, what is the rate, in kg/s, at which oxygen vapor must be vented from the system?

(b) Moisture in the ambient air will result in frost formation on the container, causing the surface emissivity to increase. Assuming the surface temperature and convection coefficient to remain at −10°C and 10 W/m^2 · K, respectively, compute the oxygen evaporation rate (kg/s) as a function of surface emissivity over the range $0.2 \le \varepsilon \le 0.94$.

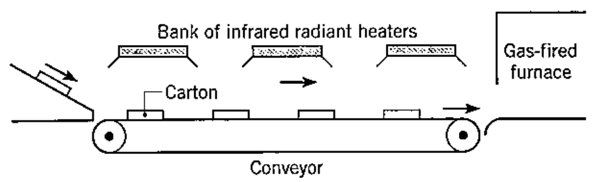
1.50 A freezer compartment is covered with a 2-mm-thick layer of frost at the time it malfunctions. If the compartment is in ambient air at 20°C and a coefficient of $h = 2\ W/m^2 \cdot K$ characterizes heat transfer by natural convection from the exposed surface of the layer, estimate the time required to completely melt the frost. The frost may be assumed to have a mass density of 700 kg/m^3 and a latent heat of fusion of 334 kJ/kg.

1.51 A vertical slab of Woods metal is joined to a substrate on one surface and is melted as it is uniformly irradiated by a laser source on the opposite surface. The metal is initially at its fusion temperature of $T_f = 72$°C, and the melt runs off by gravity as soon as it is formed. The absorptivity of the metal to the laser radiation is $\alpha_l = 0.4$, and its latent heat of fusion is $h_{sf} = 33$ kJ/kg.

(a) Neglecting heat transfer from the irradiated surface by convection or radiation exchange with the surroundings, determine the instantaneous rate of melting in kg/s · m^2 if the laser irradiation is 5 kW/m^2. How much material is removed if irradiation is maintained for a period of 2 s?

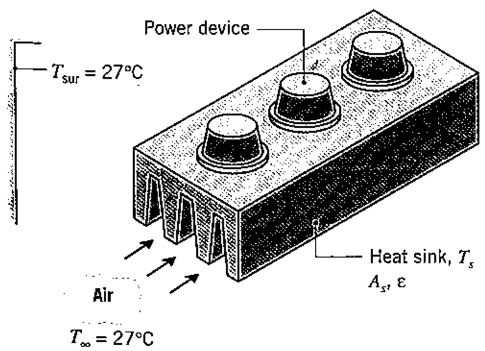
(b) Allowing for convection to ambient air, with $T_\infty = 20$°C and $h = 15\ W/m^2 \cdot K$, and radiation exchange with large surroundings ($\varepsilon = 0.4$, $T_{sur} = 20$°C), determine the instantaneous rate of melting during irradiation.

1.52 Following the hot vacuum forming of a paper–pulp mixture, the product, an egg carton, is transported on a conveyor for 18 s toward the entrance of a gas-fired oven where it is dried to a desired final water content. To increase the productivity of the line, it is proposed that a bank of infrared radiation heaters, which provide a uniform radiant flux of 5000 W/m^2, be installed over the conveyor. The carton has an exposed area of 0.0625 m^2 and a mass of 0.220 kg, 75% of which is water after the forming process.

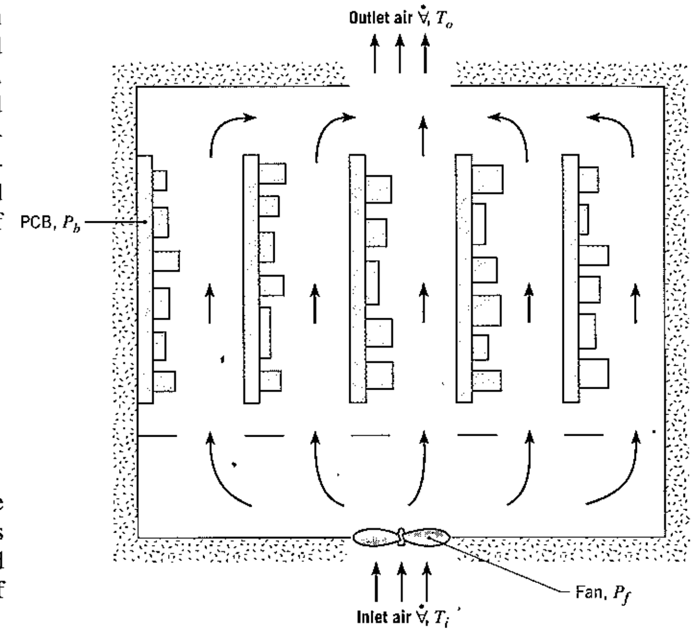


The chief engineer of your plant will approve the purchase of the heaters if the carton water content is reduced from 75% to 65%. Would you recommend the purchase? Assume the heat of vaporization of water is $h_{fg} = 2400$ kJ/kg.

1.53 Electronic power devices are mounted to a heat sink having an exposed surface area of 0.045 m^2 and an emissivity of 0.80. When the devices dissipate a total power of 20 W and the air and surroundings are at 27°C, the average sink temperature is 42°C. What average temperature will the heat sink reach when the devices dissipate 30 W for the same environmental condition?



1.54 A computer consists of an array of five printed circuit boards (PCBs), each dissipating $P_b = 20$ W of power. Cooling of the electronic components on a board is provided by the forced flow of air, equally distributed in passages formed by adjoining boards, and the convection coefficient associated with heat transfer from the components to the air is approximately $h = 200\ W/m^2 \cdot K$. Air enters the computer console at a temperature of $T_i = 20$°C, and flow is driven by a fan whose power consumption is $P_f = 25$ W.



(a) If the temperature rise of the air flow, $(T_o - T_i)$, is not to exceed 15°C, what is the minimum allowable volumetric flow rate $\dot{\forall}$ of the air? The density and specific heat of the air may be approximated as $\rho = 1.161$ kg/m^3 and $c_p = 1007$ J/kg · K, respectively.

(b) The component that is most susceptible to thermal failure dissipates 1 W/cm^2 of surface area. To minimize the potential for thermal failure, where should the component be installed on a PCB? What is its surface temperature at this location?

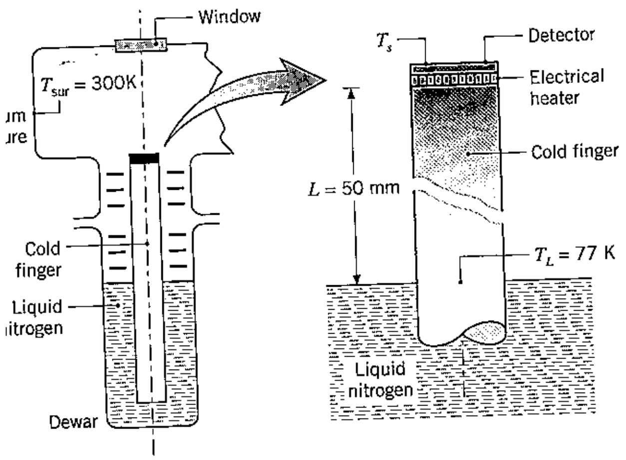
1.55 The roof of a car in a parking lot absorbs a solar radiant flux of 800 W/m^2, while the underside is perfectly insulated. The convection coefficient between the roof and the ambient air is 12 W/m^2 · K.

(a) Neglecting radiation exchange with the surroundings, calculate the temperature of the roof under steady-state conditions if the ambient air temperature is 20°C.

(b) For the same ambient air temperature, calculate the temperature of the roof if its surface emissivity is 0.8.

(c) The convection coefficient depends on airflow conditions over the roof, increasing with increasing air speed. Compute and plot the roof temperature as a function of h for $2 \le h \le 200$ $W/m^2 \cdot K$.

1.56 The operating temperature of an infrared detector for a space telescope is to be controlled by adjusting the electrical power, q_{elec}, to a thin heater sandwiched between the detector and the "cold finger" whose opposite end is immersed in liquid nitrogen at 77 K. The cold finger rod of 5-mm diameter has a thermal conductivity of 10 W/m · K and extends 50 mm above the level of the liquid nitrogen in the dewar. Assume the detector surface has an emissivity of 0.9 and the vacuum enclosure is maintained at 300 K.



(a) What is the temperature of the detector when no power is supplied to the heater?

(b) How much heater power is required to maintain the detector at 195 K?

(c) Compute and plot the heater power required to maintain a detector temperature of 195 K as a function of the thermal conductivity of the cold finger for $0.1 \le k \le 400$ W/m · K. Select a suitable finger material that would allow for maintaining the prescribed detector temperature at a low level of power consumption.

1.57 Consider the physical system depicted in Example 1.5 under conditions for which the combustion gases are at 1300°C and heat transfer by convection from the gases to the inner surface is characterized by a convection coefficient of $h_1 = 50$ $W/m^2 \cdot K$. The furnace wall is made from a diatomaceous silica brick for which $k = 0.3$ W/m · K and $\varepsilon = 0.8$, while the ambient air and surroundings remain at 25°C. Radiation exchange between the combustion gases and the inner surface may be neglected. Compute and plot the inner and outer surface temperatures, T_1 and T_2, as a function of the wall thickness ($0.025 \le L \le 0.50$ m) for an outside convection coefficient of $h_2 = 10$ $W/m^2 \cdot K$ and as a function of the convection coefficient ($2 \le h_2 \le 50$ $W/m^2 \cdot K$) for $L = 0.15$ m. Recommend values of L and h_2 suitable for maintaining T_2 below a maximum allowable value of 100°C.

1.58 For the conditions of Example 1.5, the furnace wall has inner and outer surface temperatures of $T_1 = 352$°C and $T_2 = 100$°C, respectively.

(a) To reduce the risk of burn injuries to operating personnel, the outer surface temperature should be maintained at or below 65°C. Could this objective be achieved by decreasing the wall thermal conductivity or increasing the convection coefficient by factors of two? Would the risk of burn injury be reduced by applying a low emissivity coating ($\varepsilon < 0.8$) to the outer surface?

(b) The convection coefficient at the outer surface may be varied by controlling air flow over the surface. For three different wall materials corresponding to thermal conductivities of 0.3, 0.6, and 1.2 W/m · K, plot the variation of T_2(°C) with h over the range $20 < h < 100$ $W/m^2 \cdot K$. Comment on conditions for which a safe outer surface temperature may be maintained.

1.59 A surface whose temperature is maintained at 400°C is separated from an airflow by a layer of insulation 25 mm thick for which the thermal conductivity is 0.1 W/m · K. If the air temperature is 35°C and the convection coefficient between the air and the outer surface of the insulation is 500 $W/m^2 \cdot K$, what is the temperature of this outer surface?

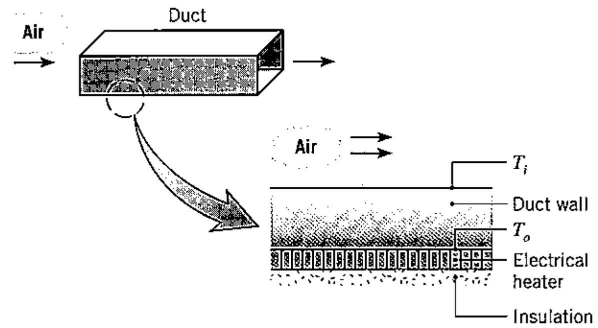
1.60 The wall of an oven used to cure plastic parts is of thickness $L = 0.05$ m and is exposed to large surroundings and air at its outer surface. The air and the surroundings are at 300 K.

(a) If the temperature of the outer surface is 400 K and its convection coefficient and emissivity are $h = 20$ $W/m^2 \cdot K$ and $\varepsilon = 0.8$, respectively, what is the temperature of the inner surface if the wall has a thermal conductivity of $k = 0.7$ W/m · K?

(b) Consider conditions for which the temperature of the inner surface is maintained at 600 K, while the air and large surroundings to which the outer surface is exposed are maintained at 300 K. Explore the effects of variations in k, h, and ε on (i) the temperature of the outer surface, (ii) the heat flux through the wall, and (iii) the heat fluxes associated with convection and radiation heat transfer from the outer surface. Specifically, compute and plot the foregoing dependent variables for parametric variations about baseline conditions of $k = 10$ W/m · K, $h = 20$ $W/m^2 \cdot K$, and $\varepsilon = 0.5$. Suggested ranges of the independent variables are $0.1 \le k \le 400$ W/m · K, $2 \le h \le 200$ $W/m^2 \cdot K$, and $0.05 \le \varepsilon \le 1$. Discuss the physical implications of your results. Under what conditions will the temperature of the outer surface be less than 45°C, which is a reasonable upper limit to avoid burn injuries if contact is made?

1.61 An experiment to determine the convection coefficient associated with airflow over the surface of a thick steel casting involves insertion of thermocouples in the casting at distances of 10 and 20 mm from the surface along a hypothetical line normal to the surface. The steel has a thermal conductivity of 15 W/m · K. If the thermocouples measure temperatures of 50 and 40°C in the steel when the air temperature is 100°C, what is the convection coefficient?

1.62 A thin electrical heating element provides a uniform heat flux q_o'' to the outer surface of a duct through which air flows. The duct wall has a thickness of 10 mm and a thermal conductivity of 20 W/m · K.



(a) At a particular location, the air temperature is 30°C and the convection heat transfer coefficient between the air and inner surface of the duct is 100 $W/m^2 \cdot K$. What heat flux q_o'' is required to maintain the inner surface of the duct at $T_i = 85$°C?

(b) For the conditions of part (a), what is the temperature (T_o) of the duct surface next to the heater?

(c) With $T_i = 85$°C, compute and plot q_o'' and T_o as a function of the air-side convection coefficient h for the range $10 \le h \le 200$ $W/m^2 \cdot K$. Briefly discuss your results.

1.63 A rectangular forced air heating duct is suspended from the ceiling of a basement whose air and walls are at a temperature of $T_\infty = T_{sur} = 5$°C. The duct is 15 m long, and its cross-section is 350 mm × 200 mm.

(a) For an uninsulated duct whose average surface temperature is 50°C, estimate the rate of heat loss from the duct. The surface emissivity and convection coefficient are approximately 0.5 and 4 $W/m^2 \cdot K$, respectively.

(b) If heated air enters the duct at 58°C and a velocity of 4 m/s and the heat loss corresponds to the result of part (a), what is the outlet temperature? The density and specific heat of the air may be assumed to be $\rho = 1.10$ kg/m^3 and $c_p = 1008$ J/kg · K, respectively.

1.64 Consider the steam pipe of Example 1.2. The facilities manager wants you to recommend methods for reducing the heat loss to the room, and two options are proposed. The first option would restrict air movement around the outer surface of the pipe and thereby reduce the convection coefficient by a factor of two. The second option would coat the outer surface of the pipe with a low emissivity ($\varepsilon = 0.4$) paint.

(a) Which of the foregoing options would you recommend?

(b) To prepare for a presentation of your recommendation to management, generate a graph of the heat loss q' as a function of the convection coefficient for $2 \le h \le 20$ $W/m^2 \cdot K$ and emissivities of 0.2, 0.4, and 0.8. Comment on the relative efficacy of reducing heat losses associated with convection and radiation.

1.65 Plate glass at 600°C is cooled by passing air over its surface such that the convection heat transfer coefficient is $h = 5$ $W/m^2 \cdot K$. To prevent cracking, it is known that the temperature gradient must not exceed 15°C/mm at any point in the glass during the cooling process. If the thermal conductivity of the glass is 1.4 W/m · K and its surface emissivity is 0.8, what is the lowest temperature of the air that can initially be used for the cooling? Assume that the temperature of the air equals that of the surroundings.

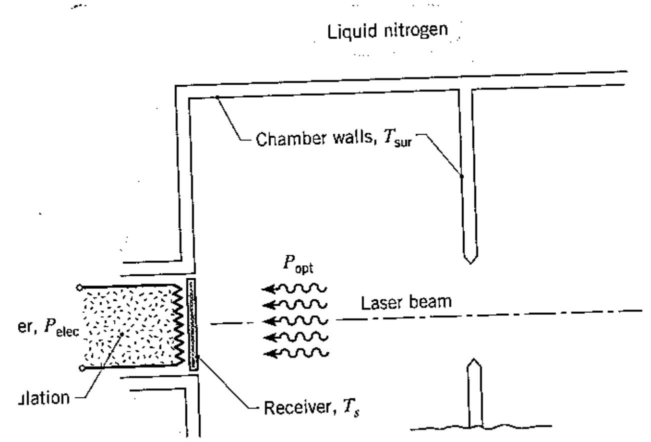
1.66 The curing process of Example 1.6 involves exposure of the plate to irradiation from an infrared lamp and attendant cooling by convection and radiation exchange with the surroundings. Alternatively, in lieu of the lamp, heating may be achieved by inserting the plate in an oven whose walls (the surroundings) are maintained at an elevated temperature.

(a) Consider conditions for which the oven walls are at 200°C, air flow over the plate is characterized by $T_\infty = 20°C$ and $h = 15$ $W/m^2 \cdot K$, and the coating has an emissivity of $\varepsilon = 0.5$. What is the temperature of the plate?

(b) For ambient air temperatures of 20, 40, and 60°C, determine the plate temperature as a function of the oven wall temperature over the range from

150 to 250°C. Plot your results, and identify conditions for which acceptable curing temperatures between 100 and 110°C may be maintained.

1.67 The electrical-substitution radiometer shown schematically determines the optical (radiant) power of a beam by measuring the electrical power required to heat the receiver to the same temperature. With a beam, such as a laser of optical power P_{opt}, incident on the receiver, its temperature, T_s, increases above that of the chamber walls held at a uniform temperature, $T_{sur} = 77$ K. With the optical beam blocked, the heater on the backside of the receiver is energized and the electrical power, P_{elec}, required to reach the same value of T_s is measured. The purpose of your analysis is to determine the relationship between the electrical and optical power, considering heat transfer processes experienced by the receiver.



Consider a radiometer with a 15-mm diameter receiver having a blackened surface with an emissivity of 0.95 and an absorptivity of 0.98 for the optical beam. When operating in the optical mode, conduction heat losses from the backside of the receiver are negligible. In the electrical mode, the loss amounts to 5% of the electrical power. What is the optical power of a beam when the indicated electrical power is 20.64 mW? What is the corresponding receiver temperature?

1.68 The diameter and surface emissivity of an electrically heated plate are $D = 300$ mm and $\varepsilon = 0.80$, respectively.

(a) Estimate the power needed to maintain a surface temperature of 200°C in a room for which the air and the walls are at 25°C. The coefficient characterizing heat transfer by natural convection depends on the surface temperature and, in units of W/m^2 · K, may be approximated by an expression of the form, $h = 0.80(T_s - T_\infty)^{1/3}$.

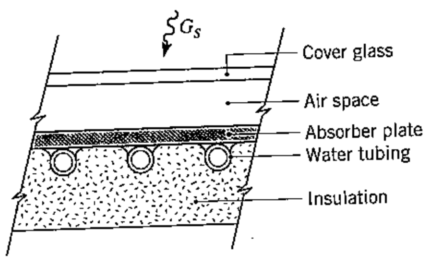
$\boxed{\text{(b)}}$ Assess the effect of surface temperature on the power requirement, as well as on the relative contributions of convection and radiation to heat transfer from the surface.

1.69 Bus bars proposed for use in a power transmission station have a rectangular cross section of height $H = 600$ mm and width $W = 200$ mm. The electrical resistivity, $\rho_e(\mu\Omega \cdot \text{m})$, of the bar material is a function of temperature, $\rho_e = \rho_{e,o}[1 + \alpha(T - T_o)]$, where $\rho_{e,o} = 0.0828$ $\mu\Omega \cdot$ m, $T_o = 25$°C and $\alpha = 0.0040$ K^{-1}. The emissivity of the bar's painted surface is 0.8, and the temperature of the surroundings is 30°C. The convection coefficient between the bar and the ambient air at 30°C is 10 W/m^2 · K.

(a) Assuming the bar has a uniform temperature T, calculate the steady-state temperature when a current of 60,000 A passes through the bar.

$\boxed{\text{(b)}}$ Compute and plot the steady-state temperature of the bar as a function of the convection coefficient for $10 \le h \le 100$ W/m^2 · K. What minimum convection coefficient is required to maintain a safe-operating temperature below 120°C? Will increasing the emissivity significantly affect this result?

1.70 A solar flux of 700 W/m^2 is incident on a flat-plate solar collector used to heat water. The area of the collector is 3 m^2, and 90% of the solar radiation passes through the cover glass and is absorbed by the absorber plate. The remaining 10% is reflected away from the collector. Water flows through the tube passages on the back side of the absorber plate and is heated from an inlet temperature T_i to an outlet temperature T_o. The cover glass, operating at a temperature of 30°C, has an emissivity of 0.94 and experiences radiation exchange with the sky at −10°C. The convection coefficient between the cover glass and the ambient air at 25°C is 10 W/m^2 · K.

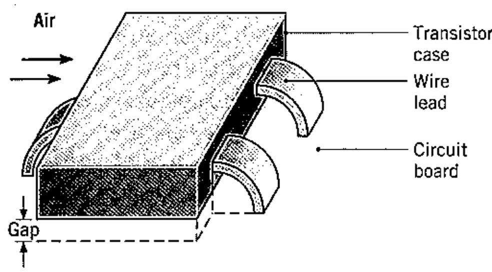


(a) Perform an overall energy balance on the collector to obtain an expression for the rate at which useful heat is collected per unit area of the collector, q''_u. Determine the value of q''_u.

(b) Calculate the temperature rise of the water, $T_o - T_i$, if the flow rate is 0.01 kg/s. Assume the specific heat of the water to be 4179 J/kg · K.

(c) The collector efficiency η is defined as the ratio of the useful heat collected to the rate at which solar energy is incident on the collector. What is the value of η?

1.71 Consider a surface-mount type transistor on a circuit board whose temperature is maintained at 35°C. Air at 20°C flows over the upper surface of dimensions 4 mm by 8 mm with a convection coefficient of 50 W/m^2 · K. Three wire leads, each of cross section 1 mm by 0.25 mm and length 4 mm, conduct heat from the case to the circuit board. The gap between the case and the board is 0.2 mm.



(a) Assuming the case is isothermal and neglecting radiation, estimate the case temperature when 150 mW are dissipated by the transistor and (i) stagnant air or (ii) a conductive paste fills the gap. The thermal conductivities of the wire leads, air, and conductive paste are 25, 0.0263, and 0.12 W/m · K, respectively.

$\boxed{\text{(b)}}$ Using the conductive paste to fill the gap, we wish to determine the extent to which increased heat dissipation may be accommodated, subject to the constraint that the case temperature not exceed 40°C. Options include increasing the air speed to achieve a larger convection coefficient h and/or changing the lead wire material to one of larger thermal conductivity. Independently considering leads fabricated from materials with thermal conductivities of 200 and 400 W/m · K, compute and plot the maximum allowable heat dissipation for variations in h over the range $50 \le h \le 250$ W/m^2 · K.

Process Identification

1.72 In analyzing the performance of a thermal system, the engineer must be able to identify the relevant heat transfer processes. Only then can the system behavior be properly quantified. For the following systems identify the pertinent processes, designating them by appropriately labeled arrows on a sketch of the system. Answer additional questions that appear in the problem statement.

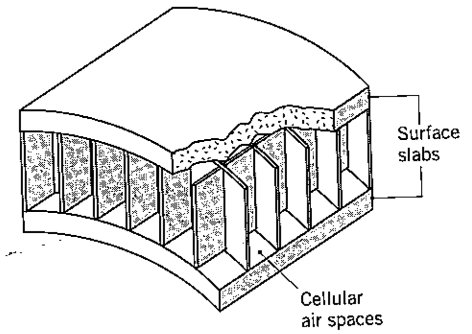
(a) Identify the heat transfer processes that determine the temperature of an asphalt pavement on a summer day. Write an energy balance for the surface of the pavement.

(b) Microwave radiation is known to be transmitted by plastics, glass, and ceramics, but to be absorbed by materials having polar molecules such as water. Water molecules exposed to microwave radiation align and reverse alignment with the microwave radiation at frequencies up to 10^9 s^{-1}, causing heat to be generated. Contrast cooking in a microwave oven with cooking in a conventional radiant or convection oven. In each case what is the physical mechanism responsible for heating the food? Which oven has the greater energy utilization efficiency? Why? Microwave heating is being considered for drying clothes. How would operation of a microwave clothes dryer differ from a conventional dryer? Which is likely to have the greater energy utilization efficiency and why?

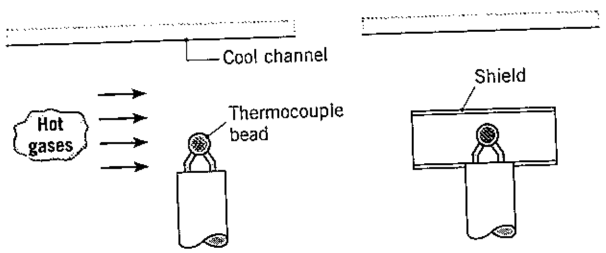
(c) Consider an exposed portion of your body (e.g., your forearm with a short-sleeved shirt) while you are sitting in a room. Identify all heat transfer processes that occur at the surface of your skin. In the interest of conserving fuel and funds, you keep the thermostat of your home at 15°C (59°F) throughout the winter months. You are able to tolerate this condition if the outside (ambient) air temperature exceeds −10°C (14°F) but feel cold if the ambient temperature falls much below this value. Are you imagining things?

(d) Consider an incandescent light source that consists of a tungsten filament enclosed in a gas-filled glass bulb. Assuming steady-state operation with the filament at a temperature of approximately 2900 K, list all the pertinent heat transfer processes for (i) the filament and (ii) the glass bulb.

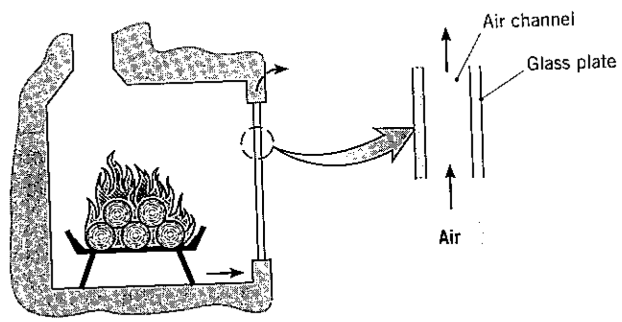
(e) There is considerable interest in developing building materials that have improved insulating qualities. The development of such materials would do much to enhance energy conservation by reducing space heating requirements. It has been suggested that superior structural and insulating qualities could be obtained by using the composite shown. The material consists of a honeycomb, with cells of square cross section, sandwiched between solid slabs. The cells are filled with air, and the slabs, as well as the honeycomb matrix, are fabricated from plastics of low thermal conductivity. Identify all heat transfer processes pertinent to the performance of the composite. Suggest ways in which this performance could be enhanced.



(f) A thermocouple junction is used to measure the temperature of a hot gas stream flowing through a channel by inserting the junction into the mainstream of the gas. The surface of the channel is cooled such that its temperature is well below that of the gas. Identify the heat transfer processes associated with the junction surface. Will the junction sense a temperature that is less than, equal to, or greater than the gas temperature? A radiation shield is a small, open-ended tube that encloses the thermocouple junction, yet allows for passage of the gas through the tube. How does use of such a shield improve the accuracy of the temperature measurement?

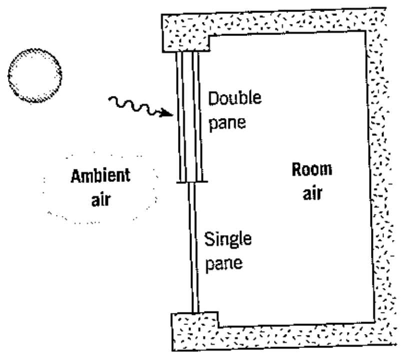


(g) A double-glazed, glass fire screen is inserted between a wood-burning fireplace and the interior of a room. The screen consists of two vertical glass plates that are separated by a space through which room air may flow (the space is open at the top and bottom). Identify the heat transfer processes associated with the fire screen.



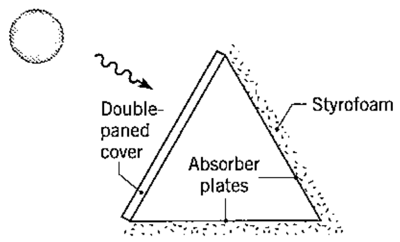
1.73 In considering the following problems involving heat transfer in the natural environment (outdoors), recognize that solar radiation is comprised of long and short wavelength components. If this radiation is incident on a *semitransparent medium,* such as water or glass, two things will happen to the nonreflected portion of the radiation. The long wavelength component will be absorbed at the surface of the medium, whereas the short wavelength component will be transmitted by the surface.

(a) The number of panes in a window can strongly influence the heat loss from a heated room to the outside ambient air. Compare the single- and double-paned units shown by identifying relevant heat transfer processes for each case.



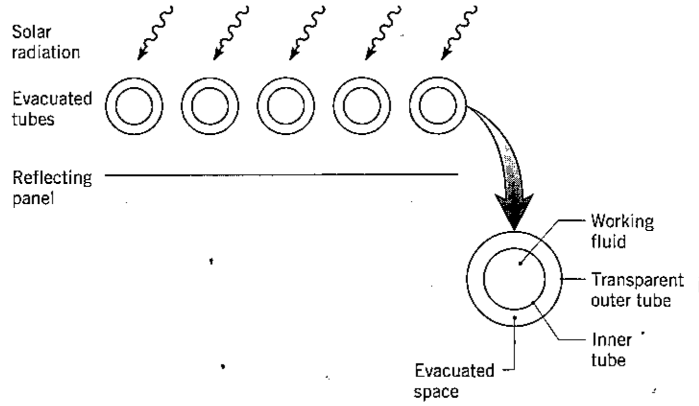
(b) In a typical flat-plate solar collector, energy is collected by a working fluid that is circulated through tubes that are in good contact with the back face of an absorber plate. The back face is insulated from the surroundings, and the absorber plate receives solar radiation on its front face, which is typically covered by one or more transparent plates. Identify the relevant heat transfer processes, first for the absorber plate with no cover plate and then for the absorber plate with a single cover plate.

(c) The solar energy collector design shown below has been used for agricultural applications. Air is blown through a long duct whose cross section is in the form of an equilateral triangle. One side of the triangle is comprised of a double-paned, semitransparent cover, while the other two sides are constructed from aluminum sheets painted flat black on the inside and covered on the outside with a layer of styrofoam insulation. During sunny periods, air entering the system is heated for delivery to either a greenhouse, grain drying unit, or a storage system.



Identify all heat transfer processes associated with the cover plates, the absorber plate(s), and the air.

(d) Evacuated-tube solar collectors are capable of improved performance relative to flat-plate collectors. The design consists of an inner tube enclosed in an outer tube that is transparent to solar radiation. The annular space between the tubes is evacuated. The outer, opaque surface of the inner tube absorbs solar radiation, and a working fluid is passed through the tube to collect the solar energy. The collector design generally consists of a row of such tubes arranged in front of a reflecting panel. Identify all heat transfer processes relevant to the performance of this device.



2.5 Summary

The primary objectives of this chapter were to improve your understanding of the conduction rate equation (Fourier's law) and to familiarize you with the heat equation. You may test your understanding of related concepts by addressing the following questions.

- In the general formulation of *Fourier's law* (applicable to any geometry), what are the vector and scalar quantities? Why is there a minus sign on the right-hand side of the equation?
- What is an *isothermal surface*? What can be said about the heat flux at any location on this surface?
- What form does *Fourier's law* take for each of the orthogonal directions of Cartesian, cylindrical and spherical coordinate systems? In each case, what are the units of the temperature gradient? Can you write each equation from memory?
- An important property of matter is defined by *Fourier's law*. What is it? What is its physical significance? What are its units?
- What is an *isotropic* material?
- Why is the thermal conductivity of a solid generally larger than that of a liquid? Why is the thermal conductivity of a liquid larger than that of a gas?
- Why is the thermal conductivity of an electrically conducting solid generally larger than that of a nonconductor? Why are materials such as beryllium oxide, diamond and silicon carbide (see Table A.2) exceptions to this rule?
- Is the *effective thermal conductivity* of an insulation system a true manifestation of the efficacy with which heat is transferred through the system by conduction?
- Why does the thermal conductivity of a gas increase with increasing temperature? Why is it approximately independent of pressure?
- What is the physical significance of the *thermal diffusivity*? How is it defined and what are its units?
- What is the physical significance of each term appearing in the *heat equation*?
- Cite some examples of *thermal energy generation*? If the rate at which thermal energy is generated per unit volume, $\dot{q}$, varies with location in a medium of volume $\forall$, how can the rate of energy generation for the entire medium, $\dot{E}_g$, be determined from knowledge of $\dot{q}(x, y, z)$?
- For a chemically reacting medium, what kind of reaction provides a *source* of thermal energy ($\dot{q} > 0$)? What kind of reaction provides a *sink* for thermal energy ($\dot{q} < 0$)?
- To solve the *heat equation* for the temperature distribution in a medium, *boundary conditions* must be prescribed at the surfaces of the medium. What physical conditions are commonly suitable for this purpose?

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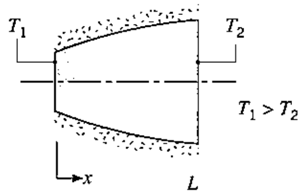
Problems

Fourier's Law

2.1 Assume steady-state, one-dimensional heat conduction through the axisymmetric shape shown below.

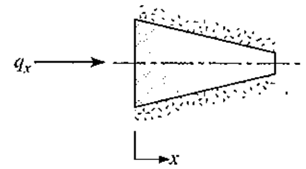


Assuming constant properties and no internal heat generation, sketch the temperature distribution on T–x coordinates. Briefly explain the shape of the curve shown.

2.2 A hot water pipe with outside radius r_1 has a temperature T_1. A thick insulation applied to reduce the heat loss has an outer radius r_2 and temperature T_2. On T–r coordinates, sketch the temperature distribution in the insulation for one-dimensional, steady-state heat transfer with constant properties. Give a brief explanation, justifying the shape of the curve shown.

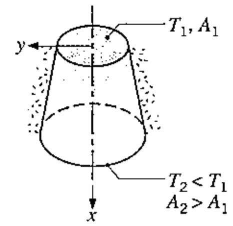
2.3 A spherical shell with inner radius r_1 and outer radius r_2 has surface temperatures T_1 and T_2, respectively, where $T_1 > T_2$. Sketch the temperature distribution on T–r coordinates assuming steady-state, one-dimensional conduction with constant properties. Briefly justify the shape of the curve shown.

2.4 Assume steady-state, one-dimensional heat conduction through the symmetric shape shown.



Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^3)$, and $q = 6000$ W, where A is in square meters, T in kelvins, and x in meters.

2.5 A solid, truncated cone serves as a support for a system that maintains the top (truncated) face of the cone at a temperature T_1, while the base of the cone is at a temperature $T_2 < T_1$.



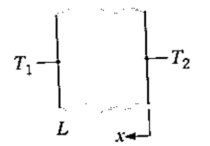
The thermal conductivity of the solid depends on temperature according to the relation $k = k_o - aT$, where a is a positive constant, and the sides of the cone are well insulated. Do the following quantities increase, decrease, or remain the same with increasing x: the heat transfer rate q_x, the heat flux q''_x, the thermal conductivity k, and the temperature gradient dT/dx?

2.6 To determine the effect of the temperature dependence of the thermal conductivity on the temperature distribution in a solid, consider a material for which this dependence may be represented as

$$k = k_o + aT$$

where k_o is a positive constant and a is a coefficient that may be positive or negative. Sketch the steady-state temperature distribution associated with heat transfer in a plane wall for three cases corresponding to $a > 0$, $a = 0$, and $a < 0$.

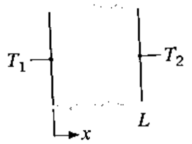
2.7 One-dimensional, steady-state conduction without heat generation occurs in the system shown. The thermal conductivity is 25 W/m · K and the thickness L is 0.5 m.



Determine the unknown quantities for each case in the accompanying table and sketch the temperature distribution, indicating the direction of the heat flux.

Case	T_1	T_2	dT/dx (K/m)	q''_x (W/m²)
1	400 K	300 K		
2	100°C		−250	
3	80°C		+200	
4		−5°C		4000
5	30°C			−3000

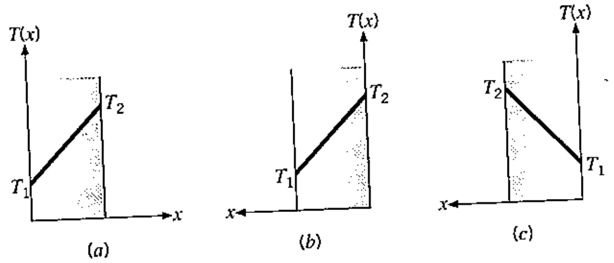
2.8 Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity $k = 50$ W/m · K and a thickness $L = 0.25$ m, with no internal heat generation.



Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.

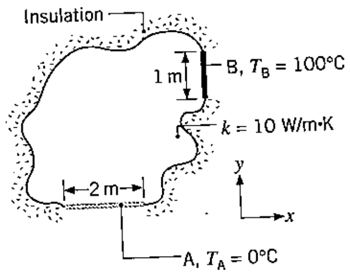
Case	T_1(°C)	T_2(°C)	dT/dx (K/m)
1	50	−20	
2	−30	−10	
3	70		160
4		40	−80
5		30	200

2.9 Consider a plane wall 100 mm thick and of thermal conductivity 100 W/m · K. Steady-state conditions are known to exist with $T_1 = 400$ K and $T_2 = 600$ K. Determine the heat flux q''_x and the temperature gradient dT/dx for the coordinate systems shown.



2.10 A cylinder of radius r_o, length L, and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞. At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at r_o and the fluid temperature.

2.11 In the two-dimensional body illustrated, the gradient at surface A is found to be $\partial T/\partial y = 30$ K/m. What are $\partial T/\partial y$ and $\partial T/\partial x$ at surface B?

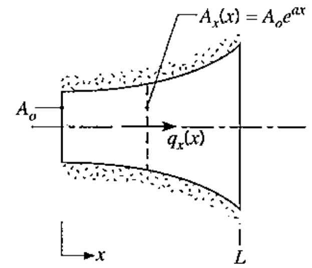


2.12 Sections of the Alaskan pipeline run above the ground and are supported by vertical steel shafts ($k = 25$ W/m · K) that are 1 m long and have a cross-sectional area of 0.005 m². Under normal operating conditions, the temperature variation along the length of a shaft is known to be governed by an expression of the form

$$T = 100 - 150x + 10x^2$$

where T and x have units of °C and meters, respectively. Temperature variations are negligible over the shaft cross section. Evaluate the temperature and conduction heat rate at the shaft–pipeline joint ($x = 0$) and at the shaft–ground interface ($x = 1$ m). Explain the difference in the heat rates.

2.13 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$, where A_o and a are constants. The lateral surface of the rod is well insulated.



(a) Write an expression for the conduction heat rate, $q_x(x)$. Use this expression to determine the temperature distribution $T(x)$ and qualitatively sketch the distribution for $T(0) > T(L)$.

(b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate $\dot{q} = \dot{q}_o \exp(-ax)$, where $\dot{q}_o$ is a constant. Obtain an expression for $q_x(x)$ when the left face ($x = 0$) is well insulated.

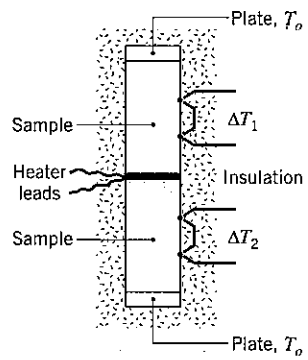
Thermophysical Properties

2.14 A solid cylindrical rod of length 0.1 m and diameter 25 mm is well insulated on its side, while its end faces are maintained at temperatures of 100 and 0°C. What is the rate of heat transfer through the rod if it is constructed from (a) pure copper, (b) aluminum alloy 2024-T6, (c) AISI 302 stainless steel, (d) silicon nitride, (e) wood (oak), (f) magnesia, 85%, and (g) Pyrex?

2.15 A one-dimensional system without heat generation has a thickness of 20 mm with surfaces maintained at temperatures of 275 and 325 K. Determine the heat flux through the system if it is constructed from (a) pure aluminum, (b) plain carbon steel, (c) AISI 316 stainless steel, (d) pyroceram, (e) Teflon, and (f) concrete.

2.16 A TV advertisement by a well-known insulation manufacturer states: it isn't the thickness of the insulating material that counts, it's the R-value. The ad shows that to obtain an R-value of 19, you need 18 ft of rock, 15 in. of wood, or just 6 in. of the manufacturer's insulation. Is this advertisement technically reasonable? If you are like most TV viewers, you don't know the R-value is defined as L/k, where L (in.) is the thickness of the insulation and k (Btu · in./hr · ft² · °F) is the thermal conductivity of the material.

2.17 An apparatus for measuring thermal conductivity employs an electrical heater sandwiched between two identical samples of diameter 30 mm and length 60 mm, which are pressed between plates maintained at a uniform temperature $T_o = 77$°C by a circulating fluid. A conducting grease is placed between all the surfaces to ensure good thermal contact. Differential thermocouples are imbedded in the samples with a spacing of 15 mm. The lateral sides of the samples are insulated to ensure one-dimensional heat transfer through the samples.

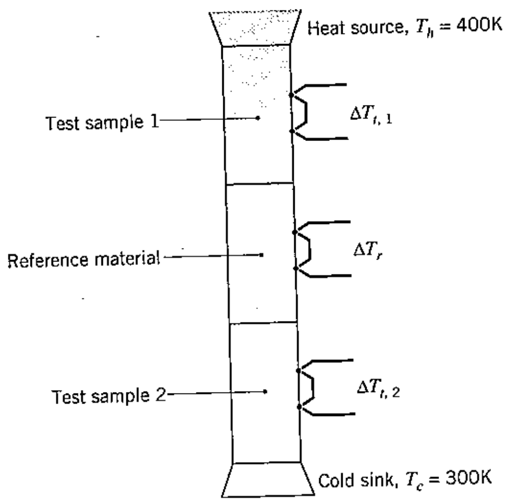


(a) With two samples of SS316 in the apparatus, the heater draws 0.353 A at 100 V and the differential thermocouples indicate $\Delta T_1 = \Delta T_2 = 25.0$°C. What is the thermal conductivity of the stainless steel sample material? What is the average temperature of the samples? Compare your result with the thermal conductivity value reported for this material in Table A.1.

(b) By mistake, an Armco iron sample is placed in the lower position of the apparatus with one of the SS316 samples from part (a) in the upper portion. For this situation, the heater draws 0.601 A at 100 V and the differential thermocouples indicate $\Delta T_1 = \Delta T_2 = 15.0°C$. What are the thermal conductivity and average temperature of the Armco iron sample?

(c) What is the advantage in constructing the apparatus with two identical samples sandwiching the heater rather than with a single heater–sample combination? When would heat leakage out of the lateral surfaces of the samples become significant? Under what conditions would you expect $\Delta T_1 \neq \Delta T_2$?

2.18 A common *comparative* method for measuring the thermal conductivity of metals is illustrated in the sketch. Cylindrical test samples (1 and 2) and a reference sample of equal diameter and length are stacked under pressure and well insulated (not shown on sketch) on their lateral surfaces. The thermal conductivity of the reference material, Armco iron in this case, is assumed known from Table A.1. For the source–sink condition of $T_h = 400$ K and $T_c = 300$ K, the differential thermocouples imbedded in the samples with a spacing of 10 mm indicate $\Delta T_r = 2.49°C$ and $\Delta T_{t,1} = \Delta T_{t,2} = 3.32°C$ for the reference and test samples, respectively.



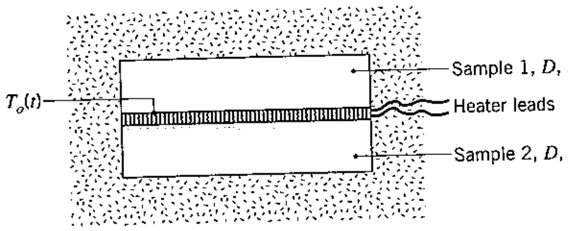
(a) What is the thermal conductivity of the test material? What temperature would you assign to this measured value?

(b) Under what conditions would you expect $\Delta T_{t,1}$ not to equal $\Delta T_{t,2}$?

2.19 A method for determining the thermal conductivity k and the specific heat c_p of a material is illustrated in the sketch. Initially the two identical samples of diameter $D = 60$ mm and thickness $L = 10$ mm and the thin heater are at a uniform temperature of $T_i = 23.00°C$, while surrounded by an insulating powder. Suddenly the heater is energized to provide a uniform heat flux q''_o on each of the sample interfaces, and the heat flux is maintained constant for a period of time, Δt_o. A short time after sudden heating is initiated, the temperature at this interface T_o is related to the heat flux as

$$T_o(t) - T_i = 2q''_o\left(\frac{t}{\pi\rho c_p k}\right)^{1/2}$$

For a particular test run, the electrical heater dissipates 15.0 W for a period of $\Delta t_o = 120$ s and the temperature at the interface is $T_o(30\text{ s}) = 24.57°C$ after 30 s of heating. A long time after the heater is deenergized, $t \gg \Delta t_o$, the samples reach the uniform temperature of $T_o(\infty) = 33.50°C$. The density of the sample materials, determined by measurement of volume and mass, is $\rho = 3965$ kg/m^3.



Determine the specific heat and thermal conductivity of the test material. By looking at values of the thermophysical properties in Table A.1 or A.2, identify the test sample material.

The Heat Equation

2.20 At a given instant of time the temperature distribution within an infinite homogeneous body is given by the function

$$T(x, y, z) = x^2 - 2y^2 + z^2 - xy + 2yz$$

Assuming constant properties and no internal heat generation, determine the regions where the temperature changes with time.

2.21 A pan is used to boil water by placing it on a stove, from which heat is transferred at a fixed rate q. There are two stages to the process. In Stage 1, the water is taken from its initial (room) temperature T_i to the boiling point, as heat is transferred from the pan by natural convection. During this stage, a constant value of the convection coefficient h may be assumed, while the bulk temperature of the water increases with time, $T_\infty = T_\infty(t)$. In Stage 2, the water has come to a boil, and its temperature remains at a fixed value, $T_\infty = T_b$, as heating continues. Consider a pan bottom of thickness L and diameter D, with a coordinate system corresponding to $x = 0$ and $x = L$ for the surfaces in contact with the stove and water, respectively.

(a) Write the form of the heat equation and the boundary/initial conditions that determine the variation of temperature with position and time, $T(x,t)$, in the pan bottom during Stage 1. Express your result in terms of the parameters q, D, L, h, and T_∞, as well as appropriate properties of the pan material.

(b) During Stage 2, the surface of the pan in contact with the water is at a fixed temperature, $T(L, t) = T_L > T_b$. Write the form of the heat equation and boundary conditions that determine the temperature distribution, $T(x)$, in the pan bottom. Express your result in terms of the parameters q, D, L, and T_L, as well as appropriate properties of the pan material.

2.22 Uniform internal heat generation at $\dot{q} = 5 \times 10^7$ W/m^3 is occurring in a cylindrical nuclear reactor fuel rod of 50-mm diameter, and under steady-state conditions the temperature distribution is of the form $T(r) = a + br^2$, where T is in degrees Celsius and r is in meters, while $a = 800°C$ and $b = -4.167 \times 10^5$ °C/m^2. The fuel rod properties are $k = 30$ W/m · K, $\rho = 1100$ kg/m^3, and $c_p = 800$ J/kg · K.

(a) What is the rate of heat transfer per unit length of the rod at $r = 0$ (the centerline) and at $r = 25$ mm (the surface)?

(b) If the reactor power level is suddenly increased to $\dot{q}_2 = 10^8$ W/m^3, what is the initial time rate of temperature change at $r = 0$ and $r = 25$ mm?

2.23 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity 50 W/m · K and thickness 50 mm is observed to be $T(°C) = a + bx^2$, where $a = 200°C$, $b = -2000$ °C/m^2, and x is in meters.

(a) What is the heat generation rate $\dot{q}$ in the wall?

(b) Determine the heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

2.24 The temperature distribution across a wall 0.3 m thick at a certain instant of time is $T(x) = a + bx + cx^2$, where T is in degrees Celsius and x is in meters, $a = 200°C$, $b = -200$ °C/m, and $c = 30$ °C/m^2. The wall has a thermal conductivity of 1 W/m · K.

(a) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

(b) If the cold surface is exposed to a fluid at 100°C, what is the convection coefficient?

2.25 A plane wall of thickness $2L = 40$ mm and thermal conductivity $k = 5$ W/m · K experiences uniform volumetric heat generation at a rate $\dot{q}$, while convection heat transfer occurs at both of its surfaces ($x = -L, +L$), each of which is exposed to a fluid of temperature $T_\infty = 20$ °C. Under steady-state conditions, the temperature distribution in the wall is of the form $T(x) = a + bx + cx^2$, where $a = 82.0°C$, $b = -210$ °C/m, $c = -2 \times 10^4$ °C/m^2, and x is in meters. The origin of the x-coordinate is at the midplane of the wall.

(a) Sketch the temperature distribution and identify significant physical features.

(b) What is the volumetric rate of heat generation $\dot{q}$ in the wall?

(c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$. How are these fluxes related to the heat generation rate?

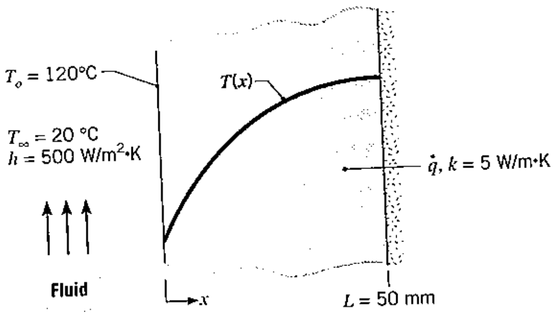
(d) What are the convection coefficients for the surfaces at $x = -L$ and $x = +L$?

(e) Obtain an expression for the heat flux distribution, $q''_x(x)$. Is the heat flux zero at any location? Explain any significant features of the distribution.

(f) If the source of the heat generation is suddenly deactivated ($\dot{q} = 0$), what is the rate of change of energy stored in the wall at this instant?

(g) What temperature will the wall eventually reach with $\dot{q} = 0$? How much energy must be removed by the fluid per unit area of the wall (J/m^2) to reach this state? The density and specific heat of the wall material are 2600 kg/m^3 and 800 J/kg · K, respectively.

2.26 One-dimensional, steady-state conduction with uniform internal energy generation occurs in a plane wall with a thickness of 50 mm and a constant thermal conductivity of 5 W/m · K. For these conditions, the temperature distribution has the form, $T(x) = a + bx + cx^2$. The surface at $x = 0$ has a temperature of $T(0) \equiv T_o = 120°C$ and experiences convection with a fluid for which $T_\infty = 20°C$ and $h = 500$ W/m^2 · K. The surface at $x = L$ is well insulated.



(a) Applying an overall energy balance to the wall, calculate the internal energy generation rate, $\dot{q}$.

(b) Determine the coefficients a, b, and c by applying the boundary conditions to the prescribed temperature distribution. Use the results to calculate and plot the temperature distribution.

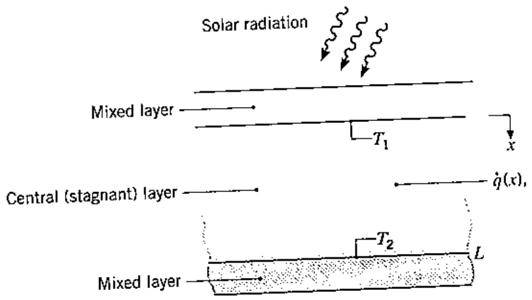
(c) Consider conditions for which the convection coefficient is halved, but the internal energy generation rate remains unchanged. Determine the new values of a, b, and c, and use the results to plot the temperature distribution. *Hint:* recognize that $T(0)$ is no longer 120°C.

(d) Under conditions for which the internal energy generation rate is doubled, and the convection coefficient remains unchanged ($h = 500\ \text{W/m}^2\cdot\text{K}$), determine the new values of a, b, and c and plot the corresponding temperature distribution. Referring to the results of Parts (b), (c), and (d) as Cases 1, 2, and 3, respectively, compare the temperature distributions for the three cases and discuss the effects of h and $\dot{q}$ on the distributions.

2.27 A salt-gradient solar pond is a shallow body of water that consists of three distinct fluid layers and is used to collect solar energy. The upper- and lower-most layers are well mixed and serve to maintain the upper and lower surfaces of the central layer at uniform temperatures T_1 and T_2, where $T_2 > T_1$. Although there is bulk fluid motion in the mixed layers, there is no such motion in the central layer. Consider conditions for which solar radiation absorption in the central layer provides nonuniform heat generation of the form $\dot{q} = Ae^{-ax}$, and the temperature distribution in the central layer is

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

The quantities A (W/m³), a (1/m), B (K/m), and C (K) are known constants having the prescribed units, and k is the thermal conductivity, which is also constant.



(a) Obtain expressions for the rate at which heat is transferred per unit area from the lower mixed layer to the central layer and from the central layer to the upper mixed layer.

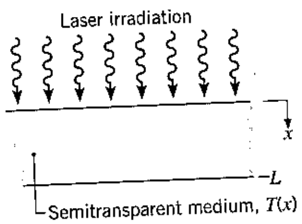
(b) Determine whether conditions are steady or transient.

(c) Obtain an expression for the rate at which thermal energy is generated in the entire central layer, per unit surface area.

2.28 The steady-state temperature distribution in a semitransparent material of thermal conductivity k and thickness L exposed to laser irradiation is of the form

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

where A, a, B, and C are known constants. For this situation, radiation absorption in the material is manifested by a distributed heat generation term, $\dot{q}(x)$.



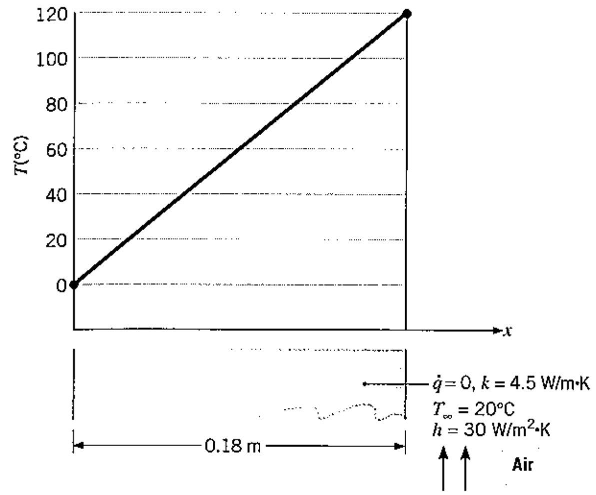
(a) Obtain expressions for the conduction heat fluxes at the front and rear surfaces.

(b) Derive an expression for $\dot{q}(x)$.

(c) Derive an expression for the rate at which radiation is absorbed in the entire material, per unit surface area. Express your result in terms of the known constants for the temperature distribution, the thermal conductivity of the material, and its thickness.

2.29 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity k and thickness L is of the form $T = ax^3 + bx^2 + cx + d$. Derive expressions for the heat generation rate per unit volume in the wall and the heat fluxes at the two wall faces ($x = 0, L$).

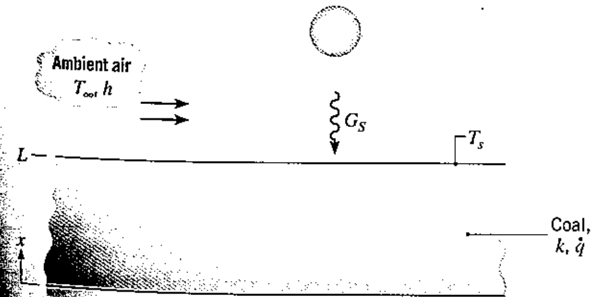
$\boxed{2.30}$ One-dimensional, steady-state conduction with no internal energy generation is occurring in a plane wall of constant thermal conductivity.



(a) Is the prescribed temperature distribution possible? Briefly explain your reasoning.

(b) With the temperature at $x = 0$ and the fluid temperature fixed at $T(0) = 0°\text{C}$ and $T_\infty = 20°\text{C}$, respectively, compute and plot the temperature at $x = L$, $T(L)$, as a function of h for $10 \le h \le 100\ \text{W/m}^2\cdot\text{K}$. Briefly explain your results.

2.31 A plane layer of coal of thickness $L = 1$ m experiences uniform volumetric generation at a rate of $\dot{q} = 20\ \text{W/m}^3$ due to slow oxidation of the coal particles. Averaged over a daily period, the top surface of the layer transfers heat by convection to ambient air for which $h = 5\ \text{W/m}^2\cdot\text{K}$ and $T_\infty = 25°\text{C}$, while receiving solar irradiation in the amount $G_S = 400\ \text{W/m}^2$. Irradiation from the atmosphere may be neglected. The solar absorptivity and emissivity of the surface are each $\alpha_S = \varepsilon = 0.95$.



(a) Write the steady-state form of the heat diffusion equation for the layer of coal. Verify that this equation is satisfied by a temperature distribution of the form

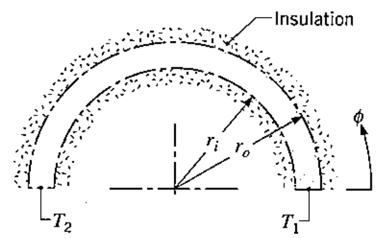
$$T(x) = T_s + \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right)$$

From this distribution, what can you say about conditions at the bottom surface ($x = 0$)? Sketch the temperature distribution and label key features.

(b) Obtain an expression for the rate of heat transfer by conduction per unit area at $x = L$. Applying an energy balance to a control surface about the top surface of the layer, obtain an expression for T_s. Evaluate T_s and $T(0)$ for the prescribed conditions.

$\boxed{\text{(c)}}$ Daily average values of G_S and h depend on a number of factors such as time of year, cloud cover, and wind conditions. For $h = 5\ \text{W/m}^2\cdot\text{K}$, compute and plot T_s and $T(0)$ as a function of G_S for $50 \le G_S \le 500\ \text{W/m}^2$. For $G_s = 400\ \text{W/m}^2$, compute and plot T_s and $T(0)$ as a function of h for $5 \le h \le 50\ \text{W/m}^2\cdot\text{K}$.

2.32 The cylindrical system illustrated has negligible variation of temperature in the r and z directions. Assume that $\Delta r = r_o - r_i$ is small compared to r_i and denote the length in the z direction, normal to the page, as L.



(a) Beginning with a properly defined control volume and considering energy generation and storage effects, derive the differential equation that prescribes the variation in temperature with the angular coordinate ϕ. Compare your result with Equation 2.20.

(b) For steady-state conditions with no internal heat generation and constant properties, determine the temperature distribution $T(\phi)$ in terms of the constants T_1, T_2, r_i, and r_o. Is this distribution linear in ϕ?

(c) For the conditions of part (b) write the expression for the heat rate q_ϕ.

2.33 Beginning with a differential control volume in the form of a cylindrical shell, derive the heat diffusion equation for a one-dimensional, cylindrical, radial coordinate system with internal heat generation. Compare your result with Equation 2.20.

2.34 Beginning with a differential control volume in the form of a spherical shell, derive the heat diffusion equation for a one-dimensional, spherical, radial coordinate system with internal heat generation. Compare your result with Equation 2.23.

2.35 Derive the heat diffusion equation, Equation 2.20, for cylindrical coordinates beginning with the differential control volume shown in Figure 2.9.

2.36 Derive the heat diffusion equation, Equation 2.23, for spherical coordinates beginning with the differential control volume shown in Figure 2.10.

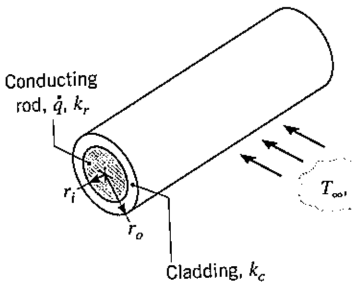
2.37 A steam pipe is wrapped with insulation of inner and outer radii, r_i and r_o, respectively. At a particular instant the temperature distribution in the insulation is known to be of the form

$$T(r) = C_1 \ln\left(\frac{r}{r_o}\right) + C_2$$

Are conditions steady-state or transient? How do the heat flux and heat rate vary with radius?

2.38 For a long circular tube of inner and outer radii r_1 and r_2, respectively, uniform temperatures T_1 and T_2 are maintained at the inner and outer surfaces, while thermal energy generation is occurring within the tube wall ($r_1 < r < r_2$). Consider steady-state conditions for which $T_1 > T_2$. Is it possible to maintain a *linear* radial temperature distribution in the wall? If so, what special conditions must exist?

2.39 Passage of an electric current through a long conducting rod of radius r_i and thermal conductivity k_r results in uniform volumetric heating at a rate of $\dot{q}$. The conducting rod is wrapped in an electrically nonconducting cladding material of outer radius r_o and thermal conductivity k_c, and convection cooling is provided by an adjoining fluid.



For steady-state conditions, write appropriate forms of the heat equations for the rod and cladding. Express appropriate boundary conditions for the solution of these equations.

2.40 Two-dimensional, steady-state conduction occurs in a hollow cylindrical solid of thermal conductivity $k = 16$ W/m · K, outer radius $r_o = 1$ m, and overall length $2z_o = 5$ m, where the origin of the coordinate system is located at the midpoint of the centerline. The inner surface of the cylinder is insulated, and the temperature distribution within the cylinder has the form $T(r, z) = a + br^2 + c \ln r + dz^2$, where $a = 20°C$, $b = 150°C/m^2$, $c = -12°C$, $d = -300°C/m^2$ and r and z are in meters

(a) Determine the inner radius r_i of the cylinder.

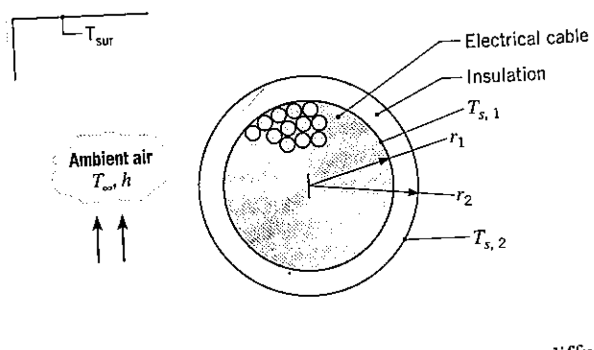
(b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}$(W/m³).

(c) Determine the axial distribution of the heat flux at the outer surface, $q''_r(r_o, z)$. What is the heat rate at the outer surface? Is it into or out of the cylinder?

(d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q''_z(r, +z_o)$ and $q''_z(r, -z_o)$. What are the corresponding heat rates? Are they into or out of the cylinder?

(e) Verify that your results are consistent with an overall energy balance on the cylinder.

2.41 An electric cable of radius r_1 and thermal conductivity k_c is enclosed by an insulating sleeve whose outer surface is of radius r_2 and experiences convection heat transfer and radiation exchange with the adjoining air and large surroundings, respectively. When electric current passes through the cable, thermal energy is generated within the cable at a volumetric rate $\dot{q}$.



(a) Write the steady-state forms of the heat diffusion equation for the insulation and the cable.

Verify that these equations are satisfied by the following temperature distributions:

Insulation: $T(r) = T_{s,2} + (T_{s,1} - T_{s,2}) \dfrac{\ln(r/r_2)}{\ln(r_1/r_2)}$

Cable: $T(r) = T_{s,1} + \dfrac{\dot{q}r_1^2}{4k_c}\left(1 - \dfrac{r^2}{r_1^2}\right)$

Sketch the temperature distribution, $T(r)$, in the cable and the sleeve, labeling key features.

(b) Applying Fourier's law, show that the rate of conduction heat transfer per unit length through the sleeve may be expressed as

$$q'_r = \frac{2\pi k_s(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

Applying an energy balance to a control surface placed around the cable, obtain an alternative expression for q'_r, expressing your result in terms of $\dot{q}$ and r_1.

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve, obtain an expression from which $T_{s,2}$ may be determined as a function of $\dot{q}$, r_1, h, T_∞, ε, and T_{sur}.

(d) Consider conditions for which 250 A are passing through a cable having an electric resistance per unit length of $R'_e = 0.005$ Ω/m, a radius of $r_1 = 15$ mm, and a thermal conductivity of $k_c = 200$ W/m · K. For $k_s = 0.15$ W/m · K, $r_2 = 15.5$ mm, $h = 25$ W/m² · K, $\varepsilon = 0.9$, $T_\infty = 25°C$, and $T_{sur} = 35°C$, evaluate the surface temperatures, $T_{s,1}$ and $T_{s,2}$, as well as the temperature T_o at the centerline of the cable.

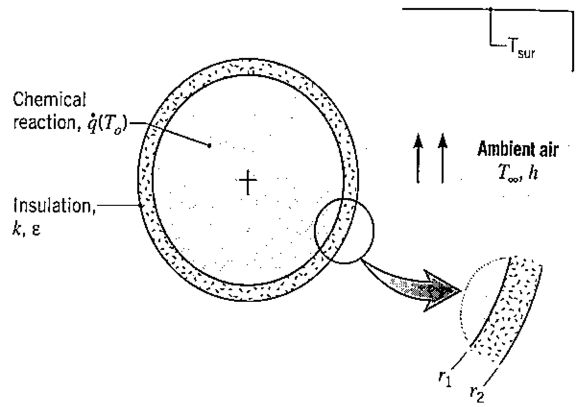
$\boxed{\text{(e)}}$ With all other conditions remaining the same, compute and plot T_o, $T_{s,1}$, and $T_{s,2}$ as a function of r_2 for $15.5 \le r_2 \le 20$ mm.

2.42 A spherical shell of inner and outer radii r_i and r_o, respectively, contains heat-dissipating components, and at a particular instant the temperature distribution in the shell is known to be of the form

$$T(r) = \frac{C_1}{r} + C_2$$

Are conditions steady-state or transient? How do the heat flux and heat rate vary with radius?

2.43 A chemically reacting mixture is stored in a thin-walled spherical container of radius $r_1 = 200$ mm, and the exothermic reaction generates heat at a uniform, but temperature-dependent volumetric rate of $\dot{q} = \dot{q}_o \exp(-A/T_o)$, where $\dot{q}_o = 5000$ W/m³, $A = 75$ K, and T_o is the mixture temperature in kelvins. The vessel is enclosed by an insulating material of outer radius r_2, thermal conductivity k, and emissivity ε. The outer surface of the insulation experiences convection heat transfer and net radiation exchange with the adjoining air and large surroundings, respectively.



(a) Write the steady-state form of the heat diffusion equation for the insulation. Verify that this equation is satisfied by the temperature distribution

$$T(r) = T_{s,1} - (T_{s,1} - T_{s,2})\left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)}\right]$$

Sketch the temperature distribution, $T(r)$, labeling key features.

(b) Applying Fourier's law, show that the rate of heat transfer by conduction through the insulation may be expressed as

$$q_r = \frac{4\pi k(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)}$$

Applying an energy balance to a control surface about the container, obtain an alternative expression for q_r, expressing your result in terms of $\dot{q}$ and r_1.

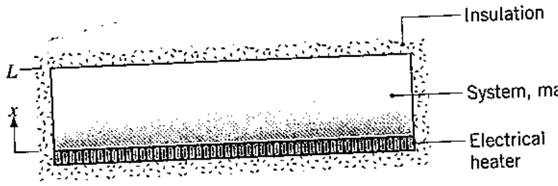
(c) Applying an energy balance to a control surface placed around the outer surface of the insulation, obtain an expression from which $T_{s,2}$ may be determined as a function of $\dot{q}$, r_1, h, T_∞, ε, and T_{sur}.

(d) The process engineer wishes to maintain a reactor temperature of $T_o = T(r_1) = 95°C$ under conditions for which $k = 0.05$ W/m · K, $r_2 = 208$ mm, $h = 5$ W/m² · K, $\varepsilon = 0.9$, $T_\infty = 25°C$, and $T_{sur} = 35°C$. What is the outer surface temperature of the insulation, $T_{s,2}$?

$\boxed{\text{(e)}}$ Compute and plot the variation of $T_{s,2}$ with r_2 for $201 \le r_2 \le 210$ mm. The engineer is concerned about potential burn injuries to personnel who may come into contact with the exposed surface of the insulation. Is increasing the insulation thickness a practical solution to maintaining $T_{s,2} \le 45°C$? What other parameter could be varied to reduce $T_{s,2}$?

Graphical Representations

2.44 The one-dimensional system of mass M with constant properties and no internal heat generation shown in the figure is initially at a uniform temperature T_i. The electrical heater is suddenly energized providing a uniform heat flux q''_o at the surface $x = 0$. The boundaries at $x = L$ and elsewhere are perfectly insulated.



(a) Write the differential equation and identify the boundary and initial conditions that could be used to determine the temperature as a function of position and time in the system.

(b) On $T-x$ coordinates, sketch the temperature distributions for the initial condition ($t \le 0$) and for several times after the heater is energized. Will a steady-state temperature distribution ever be reached?

(c) On q''_x-t coordinates, sketch the heat flux $q''_x(x,t)$ at the planes $x = 0$, $x = L/2$, and $x = L$ as a function of time.

(d) After a period of time t_e has elapsed, the heater power is switched off. Assuming that the insulation is perfect, the system will eventually reach a final uniform temperature T_f. Derive an expression that can be used to determine T_f as a function of the parameters q''_o, t_e, T_i, and the system characteristics M, c_p, and A_s (the heater surface area).

2.45 A large plate of thickness $2L$ is at a uniform temperature of $T_i = 200°C$, when it is suddenly quenched by dipping it in a liquid bath of temperature $T_\infty = 20°C$. Heat transfer to the liquid is characterized by the convection coefficient h.

(a) If $x = 0$ corresponds to the midplane of the wall, on $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \le 0$), steady-state condition ($t \to \infty$), and two intermediate times.

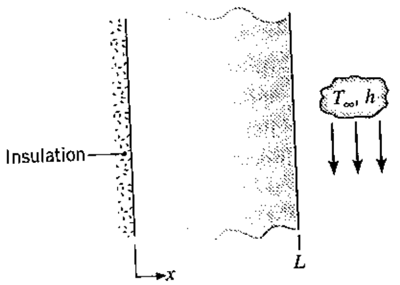
(b) On $q''_x - t$ coordinates, sketch the variation with time of the heat flux at $x = L$.

(c) If $h = 100\ W/m^2 \cdot K$, what is the heat flux at $x = L$ and $t = 0$? If the wall has a thermal conductivity of $k = 50\ W/m \cdot K$, what is the corresponding temperature gradient at $x = L$?

(d) Consider a plate of thickness $2L = 20$ mm with a density of $\rho = 2770\ kg/m^3$ and a specific heat $c_p = 875\ J/kg \cdot K$. By performing an energy balance on the plate, determine the amount of energy per unit surface area of the plate (J/m^2) that is transferred to the bath over the time required to reach steady-state conditions.

(e) From other considerations, it is known that, during the quenching process, the heat flux at $x = +L$ and $x = -L$ decays exponentially with time according to the relation, $q'' = A \exp(-Bt)$, where t is in seconds, $A = 1.80 \times 10^4\ W/m^2$, and $B = -4.126 \times 10^3\ s^{-1}$. Use this information to determine the energy per unit surface area of the plate that is transferred to the fluid during the quenching process.

2.46 The plane wall with constant properties and no internal heat generation shown in the figure is initially at a uniform temperature T_i. Suddenly the surface at $x = L$ is heated by a fluid at T_∞ having a convection heat transfer coefficient h. The boundary at $x = 0$ is perfectly insulated.



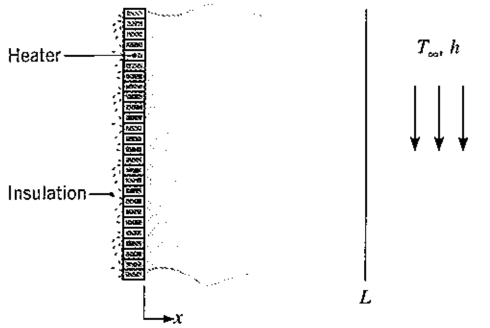
(a) Write the differential equation and identify the boundary and initial conditions that could be used to determine the temperature as a function of position and time in the wall.

(b) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \le 0$), steady-state condition ($t \to \infty$), and two intermediate times.

(c) On q''_x-t coordinates, sketch the heat flux at the locations $x = 0$ and $x = L$. That is, show qualitatively how $q''_x(0, t)$ and $q''_x(L, t)$ vary with time.

(d) Write an expression for the total energy transferred to the wall per unit volume of the wall (J/m^3).

2.47 A plane wall has constant properties, no internal heat generation, and is initially at a uniform temperature T_i. Suddenly, the surface at $x = L$ is heated by a fluid at T_∞ having a convection coefficient h. At the same instant, the electrical heater is energized providing a constant heat flux q''_o at $x = 0$.



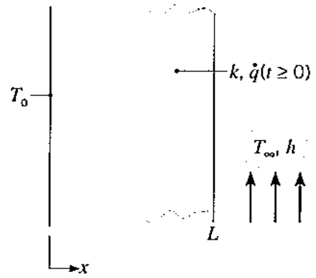
(a) On T–x coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \le 0$), steady-state condition ($t \to \infty$), and for two intermediate times.

(b) On q''_x-x coordinates, sketch the heat flux corresponding to the four temperature distributions of part (a).

(c) On q''_x-t coordinates, sketch the heat flux at the locations $x = 0$ and $x = L$. That is, show qualitatively how $q''_x(0, t)$ and $q''_x(L, t)$ vary with time.

(d) Derive an expression for the steady-state temperature at the heater surface, $T(0, \infty)$, in terms of q''_o, T_∞, k, h, and L.

2.48 A plane wall with constant properties is initially at a uniform temperature T_o. Suddenly, the surface at $x = L$ is exposed to a convection process with a fluid at T_∞ ($> T_o$) having a convection coefficient h. Also, suddenly the wall experiences a uniform internal volumetric heating $\dot{q}$ that is sufficiently large to induce a maximum steady-state temperature within the wall, which exceeds that of the fluid. The boundary at $x = 0$ remains at T_o.



(a) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \le 0$), steady-state condition ($t \to \infty$), and for two intermediate times. Show also the distribution for the special condition when there is no heat flow at the $x = L$ boundary.

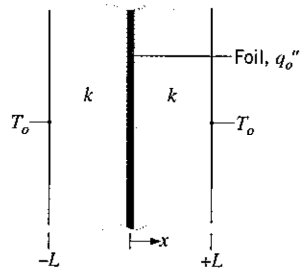
(b) On $q''_x - t$ coordinates, sketch the heat flux for the locations $x = 0$ and $x = L$, that is, $q''_x(0, t)$ and $q''_x(L, t)$, respectively.

2.49 Consider the conditions associated with Problem 2.48, but now with a convection process for which $T_\infty < T_o$.

(a) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \le 0$), steady-state condition ($t \to \infty$), and for two intermediate times. Identify key features of the distributions, especially the location of the maximum temperature and the temperature gradient at $x = L$.

(b) On $q'' - t$ coordinates, sketch the heat flux for the locations $x = 0$ and $x = L$, that is, $q''_x(0, t)$ and $q''_x(L, t)$, respectively. Identity key features of the flux histories.

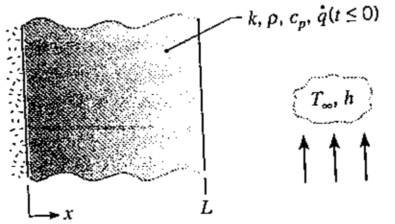
2.50 A very thin electrically conducting foil is sandwiched between two electrically nonconducting plane walls of equivalent thickness L and thermal conductivity k. If an electric current is passed through the foil, heat is generated within the foil, creating a uniform heat flux at the interface between the walls. Consider conditions for which the walls are initially at a uniform temperature T_i and Ohmic heating maintains a uniform heat flux q''_o at the interface for $t \ge 0$. Concurrently, the exposed surfaces are maintained at a fixed temperature T_o that exceeds T_i.



(a) On T–x coordinates, sketch the temperature distribution $T(x)$ in the walls $(-L \le x \le +L)$ for the initial condition $(t = 0)$, for the final steady-state condition $(t \to \infty)$, and for two intermediate times.

(b) On $q_x'' - t$ coordinates, sketch the variation of the local heat flux for the locations $x = 0$ and $x = L$, that is, $q_x''(0,t)$ and $q_x''(L,t)$, respectively.

2.51 A plane wall of thickness $L = 0.1$ m experiences uniform volumetric heating at a rate $\dot{q}$. One surface of the wall $(x = 0)$ is insulated, while the other surface is exposed to a fluid at $T_\infty = 20°C$, with convection heat transfer characterized by $h = 1000$ W/m$^2 \cdot$ K. Initially, the temperature distribution in the wall is $T(x,0) = a + bx^2$, where $a = 300°C$, $b = -1.0 \times 10^4 °C/m^2$ and x is in meters. Suddenly, the volumetric heat generation is deactivated $(\dot{q} = 0$ for $t \ge 0)$, while convection heat transfer continues to occur at $x = L$. The properties of the wall are $\rho = 7000$ kg/m^3, $c_p = 450$ J/kg $\cdot$ K, and $k = 90$ W/m $\cdot$ K.



(a) Determine the magnitude of the volumetric energy generation rate $\dot{q}$ associated with the initial condition $(t < 0)$.

(b) On $T - x$ coordinates, sketch the temperature distribution for the following conditions: initial condition $(t < 0)$, steady-state condition $(t \to \infty)$, and two intermediate conditions.

(c) On $q_x'' - t$ coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q_x''(L, t)$. Calculate the corresponding value of the heat flux at $t = 0$, $q_x''(L, 0)$.

(d) Calculate the amount of energy removed from the wall per unit area (J/m^2) by the fluid stream as the wall cools from its initial to steady-state condition.

2.52 A plane wall that is insulated on one side $(x = 0)$ is initially at a uniform temperature T_i, when its exposed surface at $x = L$ is suddenly raised to a temperature T_s.

(a) Verify that the following equation correctly characterizes the subsequent variation of the wall temperature, $T(x, t)$, with position and time:

$$\frac{T(x,t) - T_s}{T_i - T_s} = C_1 \exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

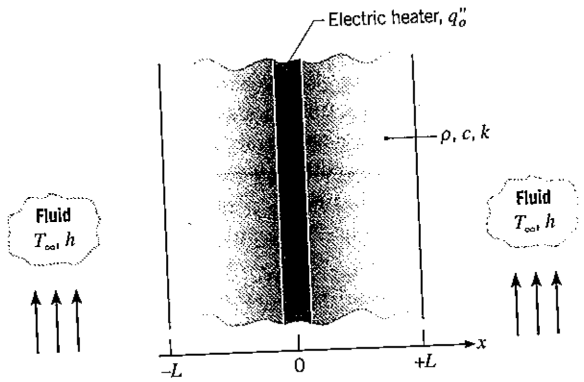
where C_1 is a constant and α is the thermal diffusivity.

(b) Obtain expressions for the heat flux at $x = 0$ and $x = L$.

(c) Sketch the temperature distribution $T(x)$ at $t = 0$, at $t \to \infty$, and at an intermediate time. Sketch the variation with time of the heat flux at $x = L$, $q_L''(t)$.

(d) What effect does α have on the thermal response of the material to a change in surface temperature?

2.53 A thin electrical heater dissipating 4000 W/m^2 is sandwiched between two 25-mm-thick plates whose exposed surfaces experience convection with a fluid for which $T_\infty = 20°C$ and $h = 400$ W/m$^2 \cdot$ K. The thermophysical properties of the plate material are $\rho = 2500$ kg/m^3, $c = 700$ J/kg$\cdot$ K, and $k = 5$ W/m$\cdot$ K.



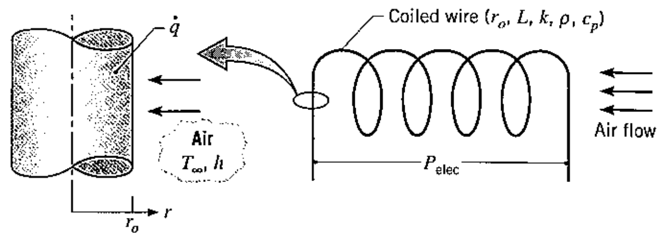
(a) On $T - x$ coordinates, sketch the steady-state temperature distribution for $-L \le x \le +L$. Calculate values of the temperatures at the surfaces, $x = \pm L$, and the midpoint, $x = 0$. Label this distribution as Case 1, and explain its salient features.

(b) Consider conditions for which there is a loss of coolant and existence of a nearly adiabatic condition on the $x = +L$ surface. On the $T - x$ coordinates used for Part (a), sketch the corresponding steady-state temperature distribution and indicate the temperatures at $x = 0, \pm L$. Label the distribution as Case 2, and explain its key features.

(c) With the system operating as described in Part (b), the surface $x = -L$ also experiences a sudden loss of coolant. This dangerous situation goes undetected for 15 minutes, at which time the power to the heater is deactivated. Assuming no heat losses from the surfaces of the plates, what is the eventual $(t \to \infty)$, uniform, steady-state temperature distribution in the plates? Show this distribution as Case 3 on your sketch, and explain its key features. *Hint*: Apply the conservation of energy requirement on a time-interval basis, Eq. 1.11b, for the initial and final conditions corresponding to Case 2 and Case 3, respectively.

(d) On $T - t$ coordinates, sketch the temperature history at the plate locations $x = 0, \pm L$ during the transient period between the distributions for Cases 2 and 3. Where and when will the temperature in the system achieve a maximum value?

2.54 Typically, air is heated in a hair dryer by blowing it across a coiled wire through which an electric current is passed. Thermal energy is generated by electric resistance heating within the wire and is transferred by convection from the surface of the wire to the air. Consider conditions for which the wire is initially at room temperature, T_i, and resistance heating is concurrently initiated with air flow at $t = 0$.



(a) For a wire radius r_o, an air temperature T_∞, and a convection coefficient h, write the form of the heat equation and the boundary/initial conditions that govern the transient thermal response, $T(r, t)$, of the wire.

(b) If the length and radius of the wire are 500 mm and 1 mm, respectively, what is the volumetric rate of thermal energy generation for a power consumption of $P_{elec} = 500$ W? What is the convection heat flux under steady-state conditions?

(c) On $T - r$ coordinates, sketch the temperature distributions for the following conditions: initial condition $(t \le 0)$, steady-state condition $(t \to \infty)$, and for two intermediate times.

(d) On $q_r'' - t$ coordinates, sketch the variation of the heat flux with time for locations at $r = 0$ and $r = r_o$.

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Problems

Plane Wall

3.1 Consider the plane wall of Figure 3.1, separating hot and cold fluids at temperatures $T_{\infty,1}$ and $T_{\infty,2}$, respectively. Using surface energy balances as boundary conditions at $x = 0$ and $x = L$ (see Equation 2.27), obtain the temperature distribution within the wall and the heat flux in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k, and L.

3.2 The rear window of an automobile is defogged by passing warm air over its inner surface.

(a) If the warm air is at $T_{\infty,i} = 40°C$ and the corresponding convection coefficient is $h_i = 30\ W/m^2 \cdot K$, what are the inner and outer surface temperatures of 4-mm-thick window glass, if the outside ambient air temperature is $T_{\infty,o} = -10°C$ and the associated convection coefficient is $h_o = 65\ W/m^2 \cdot K$?

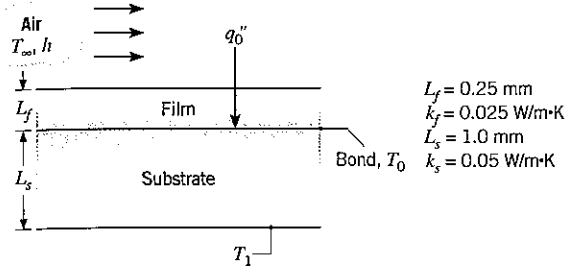
$\boxed{(b)}$ In practice $T_{\infty,o}$ and h_o vary according to weather conditions and car speed. For values of $h_o = 2$, 65, and $100\ W/m^2 \cdot K$, compute and plot the inner and outer surface temperatures as a function of $T_{\infty,o}$ for $-30 \leq T_{\infty,o} \leq 0°C$.

3.3 The rear window of an automobile is defogged by attaching a thin, transparent, film-type heating element to its inner surface. By electrically heating this element, a uniform heat flux may be established at the inner surface.

(a) For 4-mm-thick window glass, determine the electrical power required per unit window area to maintain an inner surface temperature of 15°C when the interior air temperature and convection coefficient are $T_{\infty,i} = 25°C$ and $h_i = 10\ W/m^2 \cdot K$, while the exterior (ambient) air temperature and convection coefficient are $T_{\infty,o} = -10°C$ and $h_o = 65\ W/m^2 \cdot K$.

$\boxed{(b)}$ In practice $T_{\infty,o}$ and h_o vary according to weather conditions and car speed. For values of $h_o = 2$, 20, 65, and $100\ W/m^2 \cdot K$, determine and plot the electrical power requirement as a function of $T_{\infty,o}$ for $-30 \leq T_{\infty,o} \leq 0°C$. From your results, what can you conclude about the need for heater operation at low values of h_o? How is this conclusion affected by the value of $T_{\infty,o}$? If $h \propto V^n$, where V is the vehicle speed and n is a positive exponent, how does the vehicle speed affect the need for heater operation?

3.4 In a manufacturing process, a transparent film is being bonded to a substrate as shown in the sketch. To cure the bond at a temperature T_0, a radiant source is used to provide a heat flux q''_0 (W/m²), all of which is absorbed at the bonded surface. The back of the substrate is maintained at T_1 while the free surface of the film is exposed to air at T_∞ and a convection heat transfer coefficient h.



(a) Show the thermal circuit representing the steady-state heat transfer situation. Be sure to label *all* elements, nodes, and heat rates. Leave in symbolic form.

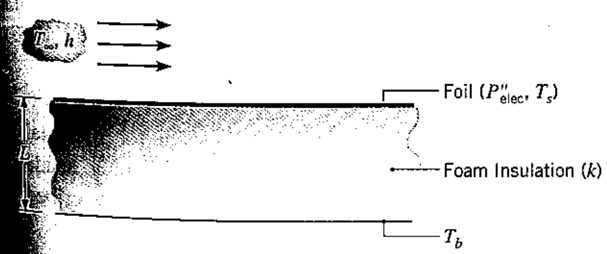
(b) Assume the following conditions: $T_\infty = 20°C$, $h = 50\ W/m^2 \cdot K$, and $T_1 = 30°C$. Calculate the heat flux q''_0 that is required to maintain the bonded surface at $T_0 = 60°C$.

$\boxed{(c)}$ Compute and plot the required heat flux as a function of the film thickness for $0 \leq L_f \leq 1$ mm.

$\boxed{(d)}$ If the film is not transparent and all of the radiant heat flux is absorbed at its upper surface, determine the heat flux required to achieve bonding. Plot your results as a function of L_f for $0 \leq L_f \leq 1$ mm.

3.5 The walls of a refrigerator are typically constructed by sandwiching a layer of insulation between sheet metal panels. Consider a wall made from fiberglass insulation of thermal conductivity $k_i = 0.046\ W/m \cdot K$ and thickness $L_i = 50$ mm and steel panels, each of thermal conductivity $k_p = 60\ W/m \cdot K$ and thickness $L_p = 3$ mm. If the wall separates refrigerated air at $T_{\infty,i} = 4°C$ from ambient air at $T_{\infty,o} = 25°C$, what is the heat gain per unit surface area? Coefficients associated with natural convection at the inner and outer surfaces may be approximated as $h_i = h_o = 5\ W/m^2 \cdot K$.

3.6 A technique for measuring convection heat transfer coefficients involves adhering one surface of a thin metallic foil to an insulating material and exposing the other surface to the fluid flow conditions of interest.



By passing an electric current through the foil, heat is dissipated uniformly within the foil and the corresponding flux, P''_{elec}, may be inferred from related voltage and current measurements. If the insulation thickness L and thermal conductivity k are known and the fluid, foil, and insulation temperatures (T_∞, T_s, T_b) are measured, the convection coefficient may be determined. Consider conditions for which $T_\infty = T_b = 25°C$, $P''_{elec} = 2000\ W/m^2$, $L = 10$ mm, and $k = 0.040\ W/m \cdot K$.

(a) With water flow over the surface, the foil temperature measurement yields $T_s = 27°C$. Determine the convection coefficient. What error would be incurred by assuming all of the dissipated power to be transferred to the water by convection?

(b) If, instead, air flows over the surface and the temperature measurement yields $T_s = 125°C$, what is the convection coefficient? The foil has an emissivity of 0.15 and is exposed to large surroundings at 25°C. What error would be incurred by assuming all of the dissipated power to be transferred to the air by convection?

$\boxed{(c)}$ Typically, heat flux gages are operated at a fixed temperature (T_s), in which case the power dissipation provides a direct measure of the convection coefficient. For $T_s = 27°C$, plot P''_{elec} as a function of h_o for $10 \leq h_o \leq 1000\ W/m^2 \cdot K$. What effect does h_o have on the error associated with neglecting conduction through the insulation?

3.7 The *wind chill*, which is experienced on a cold, windy day, is related to increased heat transfer from exposed human skin to the surrounding atmosphere. Consider a layer of fatty tissue that is 3 mm thick and whose interior surface is maintained at a temperature of 36°C. On a calm day the convection heat transfer coefficient at the outer surface is $25\ W/m^2 \cdot K$, but with 30 km/h winds it reaches $65\ W/m^2 \cdot K$. In both cases the ambient air temperature is −15°C.

(a) What is the ratio of the heat loss per unit area from the skin for the calm day to that for the windy day?

(b) What will be the skin outer surface temperature for the calm day? For the windy day?

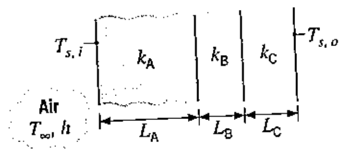
(c) What temperature would the air have to assume on the calm day to produce the same heat loss occurring with the air temperature at −15°C on the windy day?

3.8 A thermopane window consists of two pieces of glass 7 mm thick that enclose an air space 7 mm thick. The window separates room air at 20°C from outside ambient air at $-10°C$. The convection coefficient associated with the inner (room-side) surface is $10\ W/m^2 \cdot K$.

(a) If the convection coefficient associated with the outer (ambient) air is $h_o = 80\ W/m^2 \cdot K$, what is the heat loss through a window that is 0.8 m long by 0.5 m wide? Neglect radiation, and assume the air enclosed between the panes to be stagnant.

(b) Compute and plot the effect of h_o on the heat loss for $10 \leq h_o \leq 100\ W/m^2 \cdot K$. Repeat this calculation for a triple-pane construction in which a third pane and a second air space of equivalent thickness are added.

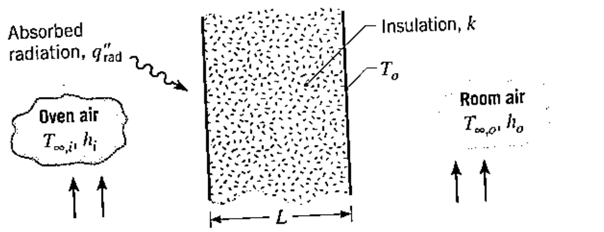
3.9 The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, $k_A = 20\ W/m \cdot K$ and $k_C = 50\ W/m \cdot K$, and known thickness, $L_A = 0.30$ m and $L_C = 0.15$ m. The third material, B, which is sandwiched between materials A and C, is of known thickness, $L_B = 0.15$ m, but unknown thermal conductivity k_B.



Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o} = 20°C$, an inner surface temperature of $T_{s,i} = 600°C$, and an oven air temperature of $T_\infty = 800°C$. The inside convection coefficient h is known to be $25\ W/m^2 \cdot K$. What is the value of k_B?

3.10 Consider the self-cleaning oven of Example 3.1. In the Comments section, it was shown that, for an oven wall temperature of $T_w = 400°C$ and an outer convection coefficient of $h_o = 30\ W/m^2 \cdot K$, the outer surface would be at a safe-to-touch temperature of $T_{s,o} = 43°C$. For oven wall temperatures of 500 and 600°C, determine the values of h_o required to maintain this safe-to-touch temperature.

3.11 The wall of a drying oven is constructed by sandwiching an insulation material of thermal conductivity $k = 0.05\ W/m \cdot K$ between thin metal sheets. The oven air is at $T_{\infty,i} = 300°C$, and the corresponding convection coefficient is $h_i = 30\ W/m^2 \cdot K$. The inner wall surface absorbs a radiant flux of $q''_{rad} = 100\ W/m^2$ from hotter objects within the oven. The room air is at $T_{\infty,o} = 25°C$, and the overall coefficient for convection and radiation from the outer surface is $h_o = 10\ W/m^2 \cdot K$.



(a) Draw the thermal circuit for the wall and label all temperatures, heat rates, and thermal resistances.

(b) What insulation thickness L is required to maintain the outer wall surface at a *safe-to-touch* temperature of $T_o = 40°C$?

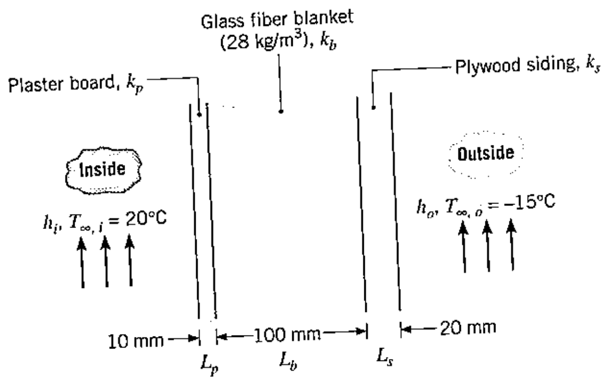
3.12 The exterior walls of a building are a composite consisting of 10-mm-thick plaster board, 50-mm-thick urethane foam, and a 10-mm-thick soft wood. On a typical winter day the outside and inside air temperatures are $-15°C$ and $20°C$, respectively, with outer and inner convection coefficients of $15\ W/m^2 \cdot K$ and $5\ W/m^2 \cdot K$, respectively.

(a) What is the heating load for a 1-m^2 section of the wall?

(b) What is the heating load if the composite wall is replaced by a 3-mm-thick glass pane window?

(c) What is the heating load if the composite wall is replaced by a double-glazed window consisting of two 3-mm-thick glass panes separated by a 5-mm-thick stagnant air gap?

3.13 A house has a composite wall of wood, fiberglass insulation, and plaster board, as indicated in the sketch. On a cold winter day the convection heat transfer coefficients are $h_o = 60\ W/m^2 \cdot K$ and $h_i = 30\ W/m^2 \cdot K$. The total wall surface area is $350\ m^2$.



(a) Determine a symbolic expression for the total thermal resistance of the wall, including inside and outside convection effects for the prescribed conditions.

(b) Determine the total heat loss through the wall.

(c) If the wind were blowing violently, raising h_o to $300\ W/m^2 \cdot K$, determine the percentage increase in the heat loss.

(d) What is the controlling resistance that determines the amount of heat flow through the wall?

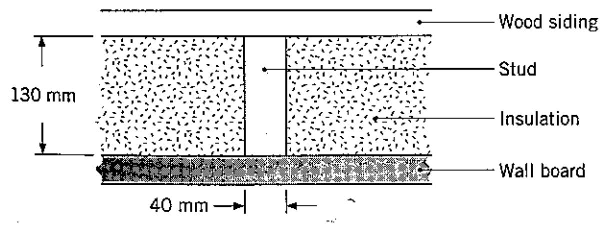
3.14 Consider the composite wall of Problem 3.13 under conditions for which the inside air is still characterized by $T_{\infty,i} = 20°C$ and $h_i = 30\ W/m^2 \cdot K$. However, use the more realistic conditions for which the outside air is characterized by a diurnal (time) varying temperature of the form

$$T_{\infty,o}(K) = 273 + 5 \sin\left(\frac{2\pi}{24} t\right) \qquad 0 \leq t \leq 12\ h$$

$$T_{\infty,o}(K) = 273 + 11 \sin\left(\frac{2\pi}{24} t\right) \qquad 12 \leq t \leq 24\ h$$

with $h_o = 60\ W/m^2 \cdot K$. Assuming quasisteady conditions for which changes in energy storage within the wall may be neglected, estimate the daily heat loss through the wall if its total surface area is $200\ m^2$.

3.15 Consider a composite wall that includes an 8-mm-thick hardwood siding, 40-mm by 130-mm hardwood studs on 0.65-m centers with glass fiber insulation (paper faced, $28\ kg/m^3$), and a 12-mm layer of gypsum (vermiculite) wall board.



What is the thermal resistance associated with a wall that is 2.5 m high by 6.5 m wide (having 10 studs, each 2.5 m high)?

3.16 The thermal characteristics of a small, dormitory refrigerator are determined by performing two separate experiments, each with the door closed and the refrigerator placed in ambient air at $T_\infty = 25°C$. In one case, an electric heater is suspended in the refrigerator cavity, while the refrigerator is unplugged. With the heater dissipating 20 W, a steady-state temperature of 90°C is recorded within the cavity. With the heater removed and the refrigerator now in operation, the second experiment involves maintaining a steady-state cavity temperature of 5°C for a fixed time interval and recording the electrical energy required to operate the refrigerator. In such an experiment for which steady operation is maintained over a 12-hour period, the input electrical energy is 125,000 J. Determine the refrigerator's coefficient of performance (COP).

3.17 In the design of buildings, energy conservation requirements dictate that the exterior surface area, A_s, be minimized. This requirement implies that, for a desired floor space, there may be optimum values associated with the number of floors and horizontal dimensions of the building. Consider a design for which the floor space, A_f, and the vertical distance between floors, H_f, are prescribed.

(a) If the building has a square cross section of width W on a side, obtain an expression for the value of W that would minimize heat loss to the surroundings. Heat loss may be assumed to occur from the four vertical side walls and from a flat roof. Express your result in terms of A_f and H_f.

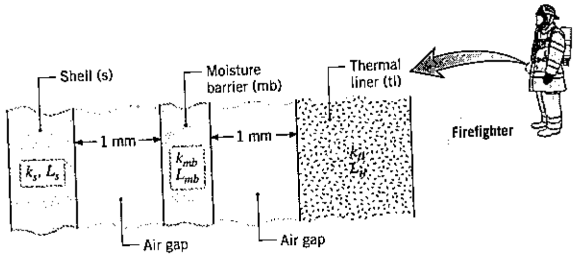
(b) If $A_f = 32{,}768\ m^2$ and $H_f = 4$ m, for what values of W and N_f (the number of floors) is the heat loss minimized? If the average overall heat transfer coefficient is $U = 1\ W/m^2 \cdot K$ and the difference between the inside and ambient air temperatures is 25°C, what is the corresponding heat loss? What is the percent reduction in heat loss compared with a building for $N_f = 2$?

3.18 Concrete of normal strength loses about 75% of its strength at approximately 600°C, and explosive spalling can occur in high-strength concrete between 350 and 600°C. Both effects can lead to structural collapse during fires. The walls of a room experiencing a *flash-over fire*, which involves all the contents of the room, may receive a radiant flux of $25\ kW/m^2$ from the fire, while also experiencing convection heat transfer with the fire-side room air at 400°C and $h = 200\ W/m^2 \cdot K$. Consider steady-state conditions for a 150-mm-thick concrete wall whose exterior surface is at 300°C, the ignition temperature for most household or office materials.

(a) Represent the wall and the heat transfer processes by a thermal circuit, labeling all the temperatures, heat fluxes, and thermal resistances.

(b) Calculate the temperature at the fire-side surface of the wall, and comment on whether the wall is likely to experience structural collapse for these conditions.

A firefighter's protective clothing, referred to as a *turnout coat,* is typically constructed as an ensemble of three layers separated by air gaps, as shown schematically.



Representative dimensions and thermal conductivities for the layers are as follows.

Layer	Thickness (mm)	k (W/m · K)
Shell (s)	0.8	0.047
Moisture barrier (mb)	0.55	0.012
Thermal liner (tl)	3.5	0.038

The air gaps between the layers are 1 mm thick, and heat is transferred by conduction and radiation exchange through the stagnant air. The linearized radiation coefficient for a gap may be approximated as, $h_{\text{rad}} = \sigma(T_1 + T_2)(T_1^2 + T_2^2) \approx 4\sigma T_{\text{avg}}^3$, where T_{avg} represents the average temperature of the surfaces comprising the gap, and the radiation flux across the gap may be expressed a $q''_{\text{rad}} = h_{\text{rad}}(T_1 - T_2)$.

(a) Represent the turnout coat by a thermal circuit, labeling all the thermal resistances. Calculate and tabulate the thermal resistances per unit area ($m^2 \cdot K/W$) for each of the layers, as well as for the conduction and radiation processes in the gaps. Assume that a value of $T_{\text{avg}} = 470$ K may be used to approximate the radiation resistance of both gaps. Comment on the relative magnitudes of the resistances.

(b) For a *pre-flash-over* fire environment in which firefighters often work, the typical radiant heat flux on the fire-side of the turnout coat is 0.25 W/cm². What is the outer surface temperature of the turnout coat if the inner surface temperature is 66°C, a condition that would result in burn injury?

Contact Resistance

3.20 A composite wall separates combustion gases at 2600°C from a liquid coolant at 100°C, with gas- and liquid-side convection coefficients of 50 and 1000 W/m² · K. The wall is composed of a 10-mm-thick layer of beryllium oxide on the gas side and a 20-mm-thick slab of stainless steel (AISI 304) on the liquid side. The contact resistance between the oxide and the steel is 0.05 m² · K/W. What is the heat loss per unit surface area of the composite? Sketch the temperature distribution from the gas to the liquid.

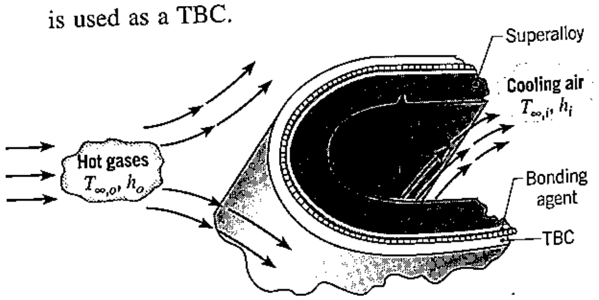
3.21 Two stainless steel plates 10 mm thick are subjected to a contact pressure of 1 bar under vacuum conditions for which there is an overall temperature drop of 100°C across the plates. What is the heat flux through the plates? What is the temperature drop across the contact plane?

3.22 Consider a plane composite wall that is composed of two materials of thermal conductivities $k_A = 0.1$ W/m · K and $k_B = 0.04$ W/m · K and thicknesses $L_A = 10$ mm and $L_B = 20$ mm. The contact resistance at the interface between the two materials is known to be 0.30 m² · K/W. Material A adjoins a fluid at 200°C for which $h = 10$ W/m² · K, and material B adjoins a fluid at 40°C for which $h = 20$ W/m² · K.

(a) What is the rate of heat transfer through a wall that is 2 m high by 2.5 m wide?

(b) Sketch the temperature distribution.

$\boxed{3.23}$ The performance of gas turbine engines may be improved by increasing the tolerance of the turbine blades to hot gases emerging from the combustor. One approach to achieving high operating temperatures involves application of a *thermal barrier coating* (TBC) to the exterior surface of a blade, while passing cooling air through the blade. Typically, the blade is made from a high-temperature superalloy, such as Inconel ($k \approx 25$ W/m · K), while a ceramic, such as zirconia ($k \approx 1.3$ W/m · K), is used as a TBC.



Consider conditions for which hot gases at $T_{\infty,o} = 1700$ K and cooling air at $T_{\infty,i} = 400$ K provide outer and inner surface convection coefficients of $h_o = 1000$ W/m² · K and $h_i = 500$ W/m² · K, respectively. If a 0.5-mm-thick zirconia TBC is attached to a 5-mm-thick Inconel blade wall by means of a metallic bonding agent, which provides an interfacial thermal resistance of $R''_{t,c} = 10^{-4}$ m² · K/W, can the Inconel be maintained at a temperature that is below its maximum allowable value of 1250 K? Radiation effects may be neglected, and the turbine blade may be approximated as a plane wall. Plot the temperature distribution with and without the TBC. Are there any limits to the thickness of the TBC?

3.24 A commercial grade cubical freezer, 3 m on a side, has a composite wall consisting of an exterior sheet of 6.35-mm-thick plain carbon steel, an intermediate layer of 100-mm-thick cork insulation, and an inner sheet of 6.35-mm-thick aluminum alloy (2024). Adhesive interfaces between the insulation and the metallic strips are each characterized by a thermal contact resistance of $R''_{t,c} = 2.5 \times 10^{-4}$ m² · K/W. What is the steady-state cooling load that must be maintained by the refrigerator under conditions for which the outer and inner surface temperatures are 22°C and −6°C respectively?

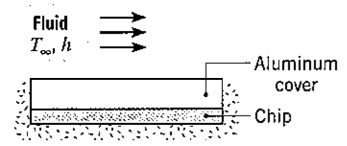
3.25 Some undergraduate students at a midwestern university have rented a poorly constructed home in which the windows are of single-pane construction. One of the students is an engineer and, like all good engineers, wishes to conserve energy, not to mention a few dollars in fuel savings. She therefore proposes that winter heat losses be reduced by covering the windows with a polystyrene insulation ($k_{\text{ins}} = 0.027$ W/m · K) during the evening hours. To estimate the energy savings, consider application of 25-mm-thick insulation panels to 6-mm-thick windows ($k_w = 1.4$ W/m · K). The contact resistance between the glass and the insulation may be approximated as ($R''_{t,c} = 0.002$ m² · K/W), while the convection coefficient at the outer surface of the window is nominally ($h_o = 20$ W/m² · K). With the insulation, the convection coefficient at the inner surface is $h_i = 2$ W/m² · K; without the insulation, it is $h_i = 5$ W/m² · K.

(a) What is the percentage reduction in heat loss associated with use of the insulation?

(b) If the total surface area of the windows in the home is $A_s = 12$ m², what are the heat losses associated with insulated and uninsulated windows for interior and exterior air temperatures of $T_{\infty,i} = 20°C$ and $T_{\infty,o} = -12°C$?

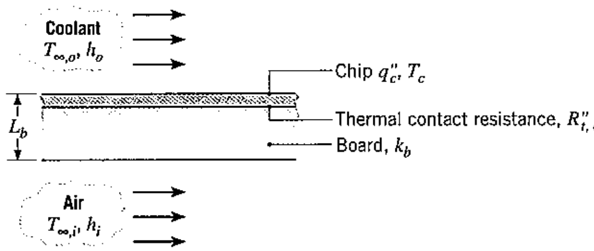
(c) If the home is heated by a gas furnace operating at an efficiency of $\eta_f = 0.80$ and natural gas is priced at $C_g = \$0.01$ per MJ, what is the daily savings associated with covering the windows for 12 hours?

3.26 A silicon chip is encapsulated such that, under steady-state conditions, all of the power it dissipates is transferred by convection to a fluid stream for which $h = 1000$ W/m² · K and $T_\infty = 25°C$. The chip is separated from the fluid by a 2-mm-thick aluminum cover plate, and the contact resistance of the chip/aluminum interface is 0.5×10^{-4} m² · K/W.



If the chip surface area is 100 mm² and its maximum allowable temperature is 85°C, what is the maximum allowable power dissipation in the chip?

3.27 Approximately 10^6 discrete electrical components can be placed on a single integrated circuit (chip), with electrical heat dissipation as high as 30,000 W/m². The chip, which is very thin, is exposed to a dielectric liquid at its outer surface, with $h_o = 1000$ W/m² · K and $T_{\infty,o} = 20°C$, and is joined to a circuit board at its inner surface. The thermal contact resistance between the chip and the board is 10^{-4} m² · K/W, and the board thickness and thermal conductivity are $L_b = 5$ mm and $k_b = 1$ W/m · K, respectively. The other surface of the board is exposed to ambient air for which $h_i = 40$ W/m² · K and $T_{\infty,i} = 20°C$.

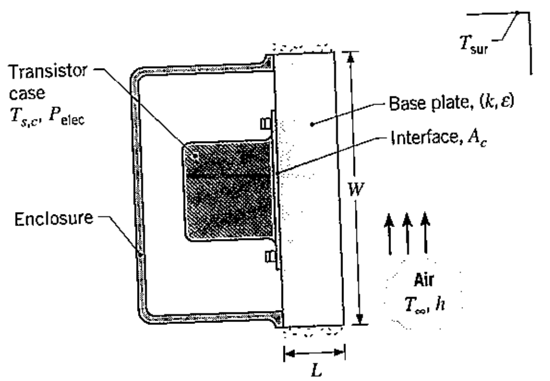


(a) Sketch the equivalent thermal circuit corresponding to steady-state conditions. In variable form, label appropriate resistances, temperatures, and heat fluxes.

(b) Under steady-state conditions for which the chip heat dissipation is $q''_c = 30{,}000$ W/m², what is the chip temperature?

$\boxed{(c)}$ The maximum allowable heat flux, $q''_{c,m}$, is determined by the constraint that the chip temperature must not exceed 85°C. Determine $q''_{c,m}$ for the foregoing conditions. If air is used in lieu of the dielectric liquid, the convection coefficient is reduced by approximately an order of magnitude. What is the value of $q''_{c,m}$ for $h_o = 100\ \text{W/m}^2\cdot\text{K}$? With air cooling, can significant improvements be realized by using an aluminum oxide circuit board and/or by using a conductive paste at the chip/board interface for which $R''_{t,c} = 10^{-5}\ \text{m}^2\cdot\text{K/W}$?

3.28 Consider a power transistor encapsulated in an aluminum case that is attached at its base to a square aluminum plate of thermal conductivity $k = 240\ \text{W/m}\cdot\text{K}$, thickness $L = 6$ mm, and width $W = 20$ mm. The case is joined to the plate by screws that maintain a contact pressure of 1 bar, and the back surface of the plate transfers heat by natural convection and radiation to ambient air and large surroundings at $T_\infty = T_{sur} = 25°\text{C}$. The surface has an emissivity of $\varepsilon = 0.9$, and the convection coefficient is $h = 4\ \text{W/m}^2\cdot\text{K}$. The case is completely enclosed such that heat transfer may be assumed to occur exclusively through the base plate.

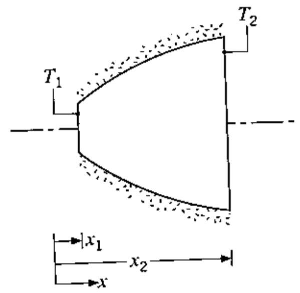


(a) If the air-filled aluminum-to-aluminum interface is characterized by an area of $A_c = 2 \times 10^{-4}\ \text{m}^2$ and a roughness of 10 μm, what is the maximum allowable power dissipation if the surface temperature of the case, $T_{s,c}$, is not to exceed 85°C?

$\boxed{\text{(b)}}$ The convection coefficient may be increased by subjecting the plate surface to a forced flow of air. Explore the effect of increasing the coefficient over the range $4 \le h \le 200\ \text{W/m}^2\cdot\text{K}$.

Alternative Conduction Analysis

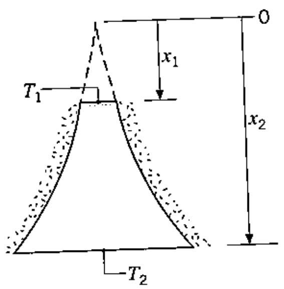
3.29 The diagram shows a conical section fabricated from pure aluminum. It is of circular cross section having diameter $D = ax^{1/2}$, where $a = 0.5\ \text{m}^{1/2}$. The small end is located at $x_1 = 25$ mm and the large end at $x_2 = 125$ mm. The end temperatures are $T_1 = 600$ K and $T_2 = 400$ K, while the lateral surface is well insulated.



(a) Derive an expression for the temperature distribution $T(x)$ in symbolic form, assuming one-dimensional conditions. Sketch the temperature distribution.

(b) Calculate the heat rate q_x.

3.30 A truncated solid cone is of circular cross section, and its diameter is related to the axial coordinate by an expression of the form $D = ax^{3/2}$, where $a = 1.0\ \text{m}^{-1/2}$.



The sides are well insulated, while the top surface of the cone at x_1 is maintained at T_1 and the bottom surface at x_2 is maintained at T_2.

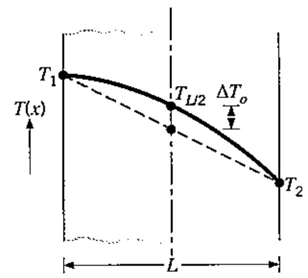
(a) Obtain an expression for the temperature distribution $T(x)$.

(b) What is the rate of heat transfer across the cone if it is constructed of pure aluminum with $x_1 = 0.075$ m, $T_1 = 100°\text{C}$, $x_2 = 0.225$ m, and $T_2 = 20°\text{C}$?

3.31 From Figure 2.5 it is evident that, over a wide temperature range, the temperature dependence of the thermal conductivity of many solids may be approximated by a linear expression of the form $k = k_o + aT$, where k_o is a positive constant and a is a coefficient that may be positive or negative. Obtain an expression for the heat flux across a plane wall whose inner and outer surfaces are maintained at T_0 and T_1, respectively. Sketch the forms of the temperature distribution corresponding to $a > 0$, $a = 0$, and $a < 0$.

3.32 Consider a tube wall of inner and outer radii r_i and r_o, whose temperatures are maintained at T_i and T_o, respectively. The thermal conductivity of the cylinder is temperature dependent and may be represented by an expression of the form $k = k_o(1 + aT)$, where k_o and a are constants. Obtain an expression for the heat transfer per unit length of the tube. What is the thermal resistance of the tube wall?

3.33 Measurements show that steady-state conduction through a plane wall without heat generation produced a convex temperature distribution such that the midpoint temperature was ΔT_o higher than expected for a linear temperature distribution.



Assuming that the thermal conductivity has a linear dependence on temperature, $k = k_o(1 + \alpha T)$, where α is a constant, develop a relationship to evaluate α in terms of ΔT_o, T_1, and T_2.

3.34 Use the alternative conduction analysis method to derive the expression for the thermal resistance of a hollow cylinder of thermal conductivity k, inner and outer radii r_i and r_o, respectively, and length L.

Cylindrical Wall

3.35 A steam pipe of 0.12-m outside diameter is insulated with a layer of calcium silicate.

(a) If the insulation is 20 mm thick and its inner and outer surfaces are maintained at $T_{s,1} = 800$ K and $T_{s,2} = 490$ K, respectively, what is the heat loss per unit length (q') of the pipe?

$\boxed{\text{(b)}}$ We wish to explore the effect of insulation thickness on the heat loss q' and outer surface temperature $T_{s,2}$, with the inner surface temperature fixed at $T_{s,1} = 800$ K. The outer surface is exposed to an airflow ($T_\infty = 25°\text{C}$) that maintains a convection coefficient of $h = 25\ \text{W/m}^2\cdot\text{K}$ and to large surroundings for which $T_{sur} = T_\infty = 25°\text{C}$. The surface emissivity of calcium silicate is approximately 0.8. Compute and plot the temperature distribution in the insulation as a function of the dimensionless radial coordinate, $(r - r_1)/(r_2 - r_1)$, where $r_1 = 0.06$ m and r_2 is a variable ($0.06 < r_2 \le 0.20$ m). Compute and plot the heat loss as a function of the insulation thickness for $0 \le (r_2 - r_1) \le 0.14$ m.

3.36 Consider the water heater described in Problem 1.37. We now wish to determine the energy needed to compensate for heat losses incurred while the water is stored at the prescribed temperature of 55°C. The cylindrical storage tank (with flat ends) has a capacity of 100 gallons, and foamed urethane is used to insulate the side and end walls from ambient air at an annual average temperature of 20°C. The resistance to heat transfer is dominated by conduction in the insulation and by free convection in the air, for which $h \approx 2\ \text{W/m}^2\cdot\text{K}$. If electric resistance heating is used to compensate for the losses and the cost of electric power is \$0.08/kWh, specify tank and insulation dimensions for which the annual cost associated with the heat losses is less than \$50.

3.37 A thin electrical heater is wrapped around the outer surface of a long cylindrical tube whose inner surface is maintained at a temperature of 5°C. The tube wall has inner and outer radii of 25 and 75 mm, respectively, and a thermal conductivity of $10\ \text{W/m}\cdot\text{K}$. The thermal contact resistance between the heater and the outer surface of the tube (per unit length of the tube) is $R'_{t,c} = 0.01\ \text{m}\cdot\text{K/W}$. The outer surface of the heater is exposed to a fluid with $T_\infty = -10°\text{C}$ and a convection coefficient of $h = 100\ \text{W/m}^2\cdot\text{K}$. Determine the heater power per unit length of tube required to maintain the heater at $T_o = 25°\text{C}$.

$\boxed{\text{3.38}}$ In the foregoing problem, the electrical power required to maintain the heater at $T_o = 25°\text{C}$ depends on the thermal conductivity of the wall material k, the thermal contact resistance $R'_{t,c}$, and the convection coefficient h. Compute and plot the separate effect of changes in k ($1 \le k \le 200\ \text{W/m}\cdot\text{K}$), $R'_{t,c}$ ($0 \le R'_{t,c} \le 0.1\ \text{m}\cdot\text{K/W}$), and h ($10 \le h \le 1000\ \text{W/m}^2\cdot\text{K}$) on the total heater power requirement, as well as the rate of heat transfer to the inner surface of the tube and to the fluid.

3.39 A stainless steel (AISI 304) tube used to transport a chilled pharmaceutical has an inner diameter of 36 mm and a wall thickness of 2 mm. The pharmaceutical and ambient air are at temperatures of 6°C and 23°C, respectively, while the corresponding inner and outer convection coefficients are 400 W/m$^2 \cdot$ K and 6 W/m$^2 \cdot$ K, respectively.

(a) What is the heat gain per unit tube length?

(b) What is the heat gain per unit length if a 10-mm-thick layer of calcium silicate insulation (k_{ins} = 0.050 W/m $\cdot$ K) is applied to the tube?

.40 Superheated steam at 575°C is routed from a boiler to the turbine of an electric power plant through steel tubes (k = 35 W/m $\cdot$ K) of 300 mm inner diameter and 30 mm wall thickness. To reduce heat loss to the surroundings and to maintain a *safe-to-touch* outer surface temperature, a layer of calcium silicate insulation (k = 0.10 W/m $\cdot$ K) is applied to the tubes, while degradation of the insulation is reduced by wrapping it in a thin sheet of aluminum having an emissivity of ε = 0.20. The air and wall temperatures of the power plant are 27°C.

(a) Assuming that the inner surface temperature of a steel tube corresponds to that of the steam and an outer convection coefficient of 6 W/m$^2 \cdot$ K for the aluminum sheet, what is the minimum insulation thickness needed to insure that the temperature of the aluminum does not exceed 50°C? What is the corresponding heat loss per meter of tube length?

$\boxed{\text{(b)}}$ Explore the effect of the insulation thickness on the temperature of the aluminum and the heat loss per unit tube length.

3.41 A thin electrical heater is inserted between a long circular rod and a concentric tube with inner and outer radii of 20 and 40 mm. The rod (A) has a thermal conductivity of k_A = 0.15 W/m $\cdot$ K, while the tube (B) has a thermal conductivity of k_B = 1.5 W/m $\cdot$ K and its outer surface is subjected to convection with a fluid of temperature T_∞ = −15°C and heat transfer coefficient 50 W/m$^2 \cdot$ K. The thermal contact resistance between the cylinder surfaces and the heater is negligible.

(a) Determine the electrical power per unit length of the cylinders (W/m) that is required to maintain the outer surface of cylinder B at 5°C.

(b) What is the temperature at the center of cylinder A?

3.42 A wire of diameter D = 2 mm and uniform temperature T has an electrical resistance of 0.01 Ω/m and a current flow of 20 A.

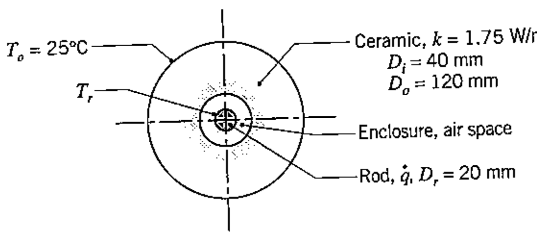
(a) What is the rate at which heat is dissipated per unit length of wire? What is the heat dissipation per unit volume within the wire?

(b) If the wire is not insulated and is in ambient air and large surrounding for which $T_\infty = T_{sur}$ = 20°C, what is the temperature T of the wire? The wire has an emissivity of 0.3, and the coefficient associated with heat transfer by natural convection may be approximated by an expression of the form, $h = C[(T - T_\infty)/D]^{1/4}$, where C = 1.25 W/m$^{7/4} \cdot$ K$^{5/4}$.

$\boxed{\text{(c)}}$ If the wire is coated with plastic insulation of 2-mm thickness and a thermal conductivity of 0.25 W/m $\cdot$ K, what are the inner and outer surface temperatures of the insulation? The insulation has an emissivity of 0.9, and the convection coefficient is given by the expression of Part (b). Explore the effect of the insulation thickness on the surface temperatures.

3.43 A 2-mm-diameter electrical wire is insulated by a 2-mm-thick rubberized sheath (k = 0.13 W/m $\cdot$ K), and the wire/sheath interface is characterized by a thermal contact resistance of $R''_{t,c} = 3 \times 10^{-4}$ m$^2 \cdot$ K/W. The convection heat transfer coefficient at the outer surface of the sheath is 10 W/m$^2 \cdot$ K, and the temperature of the ambient air is 20°C. If the temperature of the insulation may not exceed 50°C, what is the maximum allowable electrical power that may be dissipated per unit length of the conductor? What is the critical radius of the insulation?

3.44 Electric current flows through a long rod generating thermal energy at a uniform volumetric rate of $\dot{q} = 2 \times 10^6$ W/m^3. The rod is concentric with a hollow ceramic cylinder, creating an enclosure that is filled with air.



The thermal resistance per unit length due to radiation between the enclosure surfaces is R'_{rad} = 0.30 m $\cdot$ K/W, and the coefficient associated with free convection in the enclosure is h = 20 W/m$^2 \cdot$ K.

(a) Construct a thermal circuit that can be used to calculate the surface temperature of the rod, T_r. Label all temperatures, heat rates, and thermal resistances, and evaluate each thermal resistance.

(b) Calculate the surface temperature of the rod for the prescribed conditions.

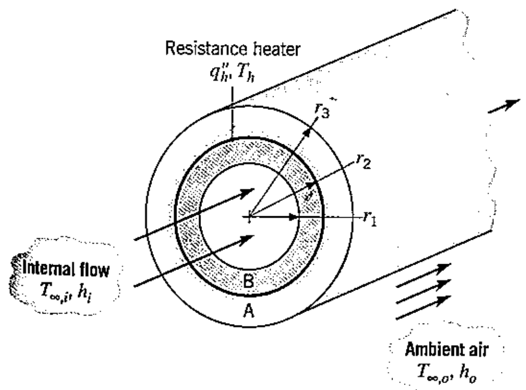
3.45 The evaporator section of a refrigeration unit consists of thin-walled, 10-mm-diameter tubes through which refrigerant passes at a temperature of −18°C. Air is cooled as it flows over the tubes, maintaining a surface convection coefficient of 100 W/m$^2 \cdot$ K, and is subsequently routed to the refrigerator compartment.

(a) For the foregoing conditions and an air temperature of −3°C, what is the rate at which heat is extracted from the air per unit tube length?

$\boxed{\text{(b)}}$ If the refrigerator's defrost unit malfunctions, frost will slowly accumulate on the outer tube surface. Assess the effect of frost formation on the cooling capacity of a tube for frost layer thicknesses in the range $0 \leq \delta \leq 4$ mm. Frost may be assumed to have a thermal conductivity of 0.4 W/m $\cdot$ K.

(c) The refrigerator is disconnected after the defrost unit malfunctions and a 2-mm-thick layer of frost has formed. If the tubes are in ambient air for which T_∞ = 20°C and natural convection maintains a convection coefficient of 2 W/m$^2 \cdot$ K, how long will it take for the frost to melt? The frost may be assumed to have a mass density of 700 kg/m^3 and a latent heat of fusion of 334 kJ/kg.

3.46 A composite cylindrical wall is composed of two materials of thermal conductivity k_A and k_B, which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible.



Liquid pumped through the tube is at a temperature $T_{\infty,i}$ and provides a convection coefficient h_i at the inner surface of the composite. The outer surface is exposed to ambient air, which is at $T_{\infty,o}$ and provides a convection coefficient of h_o. Under steady-state conditions, a uniform heat flux of q''_h is dissipated by the heater.

(a) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables.

(b) Obtain an expression that may be used to determine the heater temperature, T_h.

(c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, q'_o/q'_i. How might the variables of the problem be adjusted to minimize this ratio?

3.47 An electrical current of 700 A flows through a stainless steel cable having a diameter of 5 mm and an electrical resistance of 6×10^{-4} Ω/m (i.e., per meter of cable length). The cable is in an environment having a temperature of 30°C, and the total coefficient associated with convection and radiation between the cable and the environment is approximately 25 W/m$^2 \cdot$ K.

(a) If the cable is bare, what is its surface temperature?

(b) If a very thin coating of electrical insulation is applied to the cable, with a contact resistance of 0.02 m$^2 \cdot$ K/W, what are the insulation and cable surface temperatures?

(c) There is some concern about the ability of the insulation to withstand elevated temperatures. What thickness of this insulation (k = 0.5 W/m $\cdot$ K) will yield the lowest value of the maximum insulation temperature? What is the value of the maximum temperature when the thickness is used?

3.48 A 0.20-m diameter, thin-walled steel pipe is used to transport saturated steam at a pressure of 20 bars in a room for which the air temperature is 25°C and the convection heat transfer coefficient at the outer surface of the pipe is 20 W/m$^2 \cdot$ K.

(a) What is the heat loss per unit length from the bare pipe (no insulation)? Estimate the heat loss per unit length if a 50-mm-thick layer of insulation (magnesia, 85%) is added. The steel and magnesia may each be assumed to have an emissivity of 0.8, and the steam-side convection resistance may be neglected.

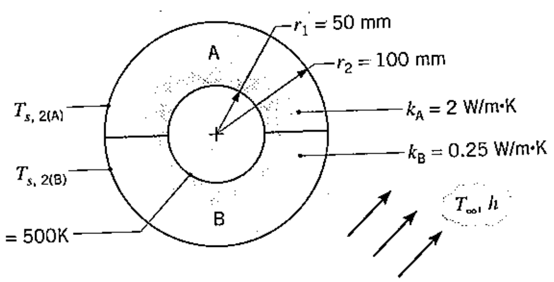
(b) The costs associated with generating the steam and installing the insulation are known to be \$4/10^9 J and \$100/m of pipe length, respectively. If the steam line is to operate 7500 h/yr, how many years are needed to pay back the initial investment in insulation?

49 Steam at a temperature of 250°C flows through a steel pipe (AISI 1010) of 60-mm inside diameter and 75-mm outside diameter. The convection coefficient between the steam and the inner surface of the pipe is 500 W/m$^2\cdot$K, while that between the outer surface of the pipe and the surroundings is 25 W/m$^2\cdot$K. The pipe emissivity is 0.8, and the temperature of the air and the surroundings is 20°C. What is the heat loss per unit length of pipe?

.50 We wish to determine the effect of adding a layer of magnesia insulation to the steam pipe of the foregoing problem. The convection coefficient at the outer surface of the insulation may be assumed to remain at 25 W/m$^2\cdot$K, and the emissivity is $\varepsilon = 0.8$. Determine and plot the heat loss per unit length of pipe and the outer surface temperature as a function of insulation thickness. If the cost of generating the steam is \$4/10^9 J and the steam line operates 7000 h/yr, recommend an insulation thickness and determine the corresponding annual savings in energy costs. Plot the temperature distribution in the insulation for the recommended thickness.

3.51 An uninsulated, thin-walled pipe of 100-mm diameter is used to transport water to equipment that operates outdoors and uses the water as a coolant. During particularly harsh winter conditions, the pipe wall achieves a temperature of -15°C and a cylindrical layer of ice forms on the inner surface of the wall. If the mean water temperature is 3°C and a convection coefficient of 2000 W/m$^2\cdot$K is maintained at the inner surface of the ice, which is at 0°C, what is the thickness of the ice layer?

3.52 Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B.



The interface between the two materials may be assumed to have an infinite contact resistance, and the entire outer surface is exposed to air for which $T_\infty = 300$ K and $h = 25$ W/m$^2\cdot$K.

(a) Sketch the thermal circuit of the system. Label (using the above symbols) all pertinent nodes and resistances.

(b) For the prescribed conditions, what is the total heat loss from the pipe? What are the outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$?

3.53 A Bakelite coating is to be used with a 10-mm-diameter conducting rod, whose surface is maintained at 200°C by passage of an electrical current. The rod is in a fluid at 25°C, and the convection coefficient is 140 W/m$^2\cdot$K. What is the critical radius associated with the coating? What is the heat transfer rate per unit length for the bare rod and for the rod with a coating of Bakelite that corresponds to the critical radius? How much Bakelite should be added to reduce the heat transfer associated with the bare rod by 25%?

Spherical Wall

3.54 A storage tank consists of a cylindrical section that has a length and inner diameter of $L = 2$ m and $D_i = 1$ m, respectively, and two hemispherical end sections. The tank is constructed from 20-mm-thick glass (Pyrex) and is exposed to ambient air for which the temperature is 300 K and the convection coefficient is 10 W/m$^2\cdot$K. The tank is used to store heated oil, which maintains the inner surface at a temperature of 400 K. Determine the electrical power that must be supplied to a heater submerged in the oil if the prescribed conditions are to be maintained. Radiation effects may be neglected, and the Pyrex may be assumed to have a thermal conductivity of 1.4 W/m·K.

3.55 Consider the liquid oxygen storage system and the laboratory environmental conditions of Problem 1.49. To reduce oxygen loss due to vaporization, an insulating layer should be applied to the outer surface of the container. Consider using a laminated aluminum foil/glass mat insulation, for which the thermal conductivity and surface emissivity are $k = 0.00016$ W/m·K and $\varepsilon = 0.20$, respectively.

(a) If the container is covered with a 10-mm-thick layer of insulation, what is the percentage reduction in oxygen loss relative to the uncovered container?

(b) Compute and plot the oxygen evaporation rate (kg/s) as a function of the insulation thickness t for $0 \le t \le 50$ mm.

3.56 In Example 3.4, an expression was derived for the critical insulation radius of an insulated, cylindrical tube. Derive the expression that would be appropriate for an insulated sphere.

3.57 A hollow aluminum sphere, with an electrical heater in the center, is used in tests to determine the thermal conductivity of insulating materials. The inner and outer radii of the sphere are 0.15 and 0.18 m, respectively, and testing is done under steady-state conditions with the inner surface of the aluminum maintained at 250°C. In a particular test, a spherical shell of insulation is cast on the outer surface of the sphere to a thickness of 0.12 m. The system is in a room for which the air temperature is 20°C and the convection coefficient at the outer surface of the insulation is 30 W/m$^2\cdot$K. If 80 W are dissipated by the heater under steady-state conditions, what is the thermal conductivity of the insulation?

3.58 A spherical tank for storing liquid oxygen on the space shuttle is to be made from stainless steel of 0.80-m outer diameter and 5-mm wall thickness. The boiling point and latent heat of vaporization of liquid oxygen are 90 K and 213 kJ/kg, respectively. The tank is to be installed in a large compartment whose temperature is to be maintained at 240 K. Design a thermal insulation system that will maintain oxygen losses due to boiling below 1 kg/day.

3.59 A spherical, cryosurgical probe may be imbedded in diseased tissue for the purpose of freezing, and thereby destroying, the tissue. Consider a probe of 3-mm diameter whose surface is maintained at -30°C when imbedded in tissue that is at 37°C. A spherical layer of frozen tissue forms around the probe, with a temperature of 0°C existing at the phase front (interface) between the frozen and normal tissue. If the thermal conductivity of frozen tissue is approximately 1.5 W/m·K and heat transfer at the phase front may be characterized by an effective convection coefficient of 50 W/m$^2\cdot$K, what is the thickness of the layer of frozen tissue?

3.60 A spherical vessel used as a reactor for producing pharmaceuticals has a 10-mm thick stainless steel wall ($k = 17$ W/m·K) and an inner diameter of 1 m. The exterior surface of the vessel is exposed to ambient air ($T_\infty = 25$°C) for which a convection coefficient of 6 W/m$^2\cdot$K may be assumed.

(a) During steady-state operation, an inner surface temperature of 50°C is maintained by energy generated within the reactor. What is the heat loss from the vessel?

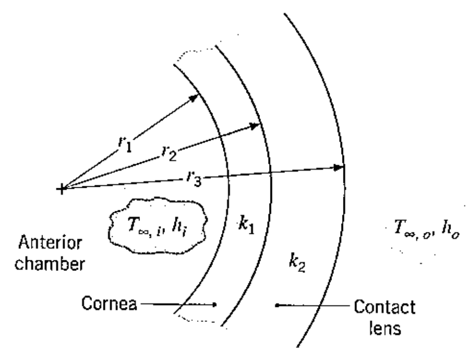
(b) If a 20-mm-thick layer of fiberglass insulation ($k = 0.040$ W/m·K) is applied to the exterior of the vessel and the rate of thermal energy generation is unchanged, what is the inner surface temperature of the vessel?

3.61 The wall of a spherical tank of 1-m diameter contains an exothermic chemical reaction and is at 200°C when the ambient air temperature is 25°C. What thickness of urethane foam is required to reduce the exterior temperature to 40°C, assuming the convection coefficient is 20 W/m$^2\cdot$K for both situations? What is the percentage reduction in heat rate achieved by using the insulation?

3.62 A composite spherical shell of inner radius $r_1 = 0.25$ m is constructed from lead of outer radius $r_2 = 0.30$ m and AISI 302 stainless steel of outer radius $r_3 = 0.31$ m. The cavity is filled with radioactive wastes that generate heat at a rate of $\dot{q} = 5 \times 10^5$ W/m^3. It is proposed to submerge the container in oceanic waters that are at a temperature of $T_\infty = 10$°C and provide a uniform convection coefficient of $h = 500$ W/m$^2\cdot$K at the outer surface of the container. Are there any problems associated with this proposal?

3.63 As an alternative to storing radioactive materials in oceanic waters, it is proposed that the system of Problem 3.62 be placed in a large tank for which the flow of water, and hence the convection coefficient h, can be controlled. Compute and plot the maximum temperature of the lead, $T(r_1)$, as a function of h for $100 \le h \le 1000$ W/m$^2\cdot$K. If the temperature of the lead is not to exceed 500 K, what is the minimum allowable value of h? To improve system reliability, it is desirable to increase the thickness of the stainless steel shell. For h = 300, 500, and 1000 W/m$^2\cdot$K, compute and plot the maximum lead temperature as a function of shell thickness for $r_3 \ge 0.30$ m. What are the corresponding values of the maximum allowable thickness?

3.64 The energy transferred from the anterior chamber of the eye through the cornea varies considerably depending on whether a contact lens is worn. Treat the eye as a spherical system and assume the system to be at steady state. The convection coefficient h_o is unchanged with and without the contact lens in place. The cornea and the lens cover one-third of the spherical surface area.



Values of the parameters representing this situation are as follows:

$r_1 = 10.2$ mm	$r_2 = 12.7$ mm
$r_3 = 16.5$ mm	
$T_{\infty,i} = 37°C$	$T_{\infty,o} = 21°C$
$k_1 = 0.35$ W/m · K	$k_2 = 0.80$ W/m · K
$h_i = 12$ W/m^2 · K	$h_o = 6$ W/m^2 · K

(a) Construct the thermal circuits, labeling all potentials and flows for the systems excluding the contact lens and including the contact lens. Write resistance elements in terms of appropriate parameters.

(b) Determine the heat loss from the anterior chamber with and without the contact lens in place.

(c) Discuss the implication of your results.

3.65 The outer surface of a hollow sphere of radius r_2 is subjected to a uniform heat flux q_2''. The inner surface at r_1 is held at a constant temperature $T_{s,1}$.

(a) Develop an expression for the temperature distribution $T(r)$ in the sphere wall in terms of q_2'', $T_{s,1}$, r_1, r_2, and the thermal conductivity of the wall material k.

(b) If the inner and outer tube radii are $r_1 = 50$ mm and $r_2 = 100$ mm, what heat flux q_2'' is required to maintain the outer surface at $T_{s,2} = 50°C$, while the inner surface is at $T_{s,1} = 20°C$? The thermal conductivity of the wall material is $k = 10$ W/m · K.

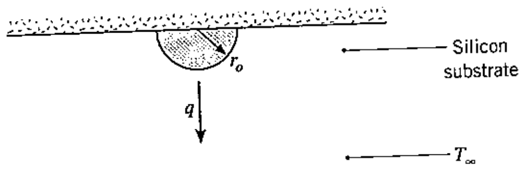
3.66 A spherical shell of inner and outer radii r_i and r_o, respectively, is filled with a heat-generating material that provides for a uniform volumetric generation rate (W/m^3) of $\dot{q}$. The outer surface of the shell is exposed to a fluid having a temperature T_∞ and a convection coefficient h. Obtain an expression for the steady-state temperature distribution $T(r)$ in the shell, expressing your result in terms of r_i, r_o, $\dot{q}$, h, T_∞, and the thermal conductivity k of the shell material.

3.67 A spherical tank of 3-m diameter contains a liquified-petroleum gas at −60°C. Insulation with a thermal conductivity of 0.06 W/m · K and thickness 250 mm is applied to the tank to reduce the heat gain.

(a) Determine the radial position in the insulation layer at which the temperature is 0°C when the ambient air temperature is 20°C and the convection coefficient on the outer surface is 6 W/m^2 · K.

(b) If the insulation is pervious to moisture from the atmospheric air, what conclusions can you reach about the formation of ice in the insulation? What effect will ice formation have on heat gain to the LP gas? How could this situation be avoided?

3.68 A transistor, which may be approximated as a hemispherical heat source of radius $r_o = 0.1$ mm, is embedded in a large silicon substrate ($k = 125$ W/m · K) and dissipates heat at a rate q. All boundaries of the silicon are maintained at an ambient temperature of $T_\infty = 27°C$, except for a plane surface that is well insulated.



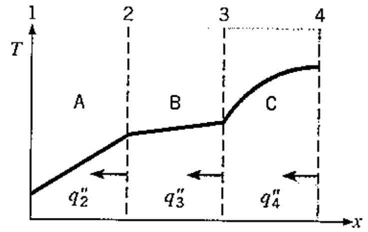
Obtain a general expression for the substrate temperature distribution and evaluate the surface temperature of the heat source for $q = 4$ W.

3.69 One modality for destroying malignant tissue involves imbedding a small spherical heat source of radius r_o within the tissue and maintaining local temperatures above a critical value T_c for an extended period. Tissue that is well removed from the source may be assumed to remain at normal body temperature ($T_b = 37°C$). Obtain a general expression for the radial temperature distribution in the tissue under steady-state conditions for which heat is dissipated at a rate q. If $r_o = 0.5$ mm, what heat rate must be supplied to maintain a tissue temperature of $T \geq T_c = 42°C$ in the domain $0.5 \leq r \leq 5$ mm? The tissue thermal conductivity is approximately 0.5 W/m · K.

Conduction with Thermal Energy Generation

3.70 Consider cylindrical and spherical shells with inner and outer surfaces at r_1 and r_2 maintained at uniform temperatures $T_{s,1}$ and $T_{s,2}$, respectively. If there is uniform heat generation within the shells, obtain expressions for the steady-state, one-dimensional radial distributions of the temperature, heat flux, and heat rate. Contrast your results with those summarized in Appendix C.

3.71 The steady-state temperature distribution in a composite plane wall of three different materials, each of constant thermal conductivity, is shown as follows.



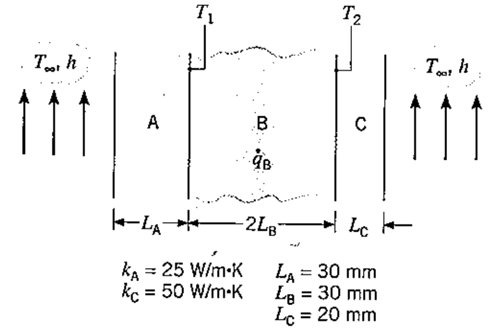
(a) Comment on the relative magnitudes of q_2'' and q_3'' and of q_3'' and q_4''.

(b) Comment on the relative magnitudes of k_A and k_B and of k_B and k_C.

(c) Sketch the heat flux as a function of x.

3.72 A plane wall of thickness 0.1 m and thermal conductivity 25 W/m · K having uniform volumetric heat generation of 0.3 MW/m^3 is insulated on one side, while the other side is exposed to a fluid at 92°C. The convection heat transfer coefficient between the wall and the fluid is 500 W/m^2 · K. Determine the maximum temperature in the wall.

3.73 Consider one-dimensional conduction in a plane composite wall. The outer surfaces are exposed to a fluid at 25°C and a convection heat transfer coefficient of 1000 W/m^2 · K. The middle wall B experiences uniform heat generation $\dot{q}_B$, while there is no generation in walls A and C. The temperatures at the interfaces are $T_1 = 261°C$ and $T_2 = 211°C$.



(a) Assuming negligible contact resistance at the interfaces, determine the volumetric heat generation $\dot{q}_B$ and the thermal conductivity k_B.

$\boxed{\text{(b)}}$ Plot the temperature distribution, showing its important features.

$\boxed{\text{(c)}}$ Consider conditions corresponding to a *loss of coolant* at the exposed surface of material A ($h = 0$). Determine T_1 and T_2 and plot the temperature distribution throughout the system.

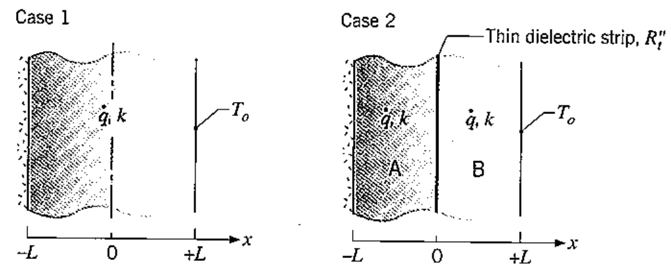
3.74 Consider the composite plane wall of Problem 3.73 subjected to the same convection conditions. The middle wall has a thermal conductivity of $k_B = 15$ W/m · K and experiences a uniform heat generation of $\dot{q}_B = 4 \times 10^6$ W/m^3, while the outer walls have no generation.

(a) Neglecting interfacial contact resistances, determine T_1 and T_2, as well as the heat fluxes through walls A and C.

$\boxed{\text{(b)}}$ Consider conditions for which contact resistances of 0.0025 and 0.001 m^2 · K/W exist at the A/B and B/C interfaces, respectively. Determine T_1 and T_2 and plot the temperature distribution.

3.75 Consider the composite wall of Example 3.6. In the Comments section, temperature distributions in the wall were determined assuming negligible contact resistance between materials A and B. Compute and plot the temperature distributions if the thermal contact resistance is $R''_{t,c} = 10^{-4}$ m^2 · K/W.

3.76 A plane wall of thickness $2L$ and thermal conductivity k experiences a uniform volumetric generation rate $\dot{q}$. As shown in the sketch for case 1, the surface at $x = -L$ is perfectly insulated, while the other surface is maintained at a uniform, constant temperature T_o. For case 2, a very thin dielectric strip is inserted at the midpoint of the wall ($x = 0$) in order to electrically isolate the two sections, A and B. The thermal resistance of the strip is $R_t'' = 0.0005$ m^2 · K/W. The parameters associated with the wall are $k = 50$ W/m · K, $L = 20$ mm, $\dot{q} = 5 \times 10^6$ W/m^3, and $T_o = 50°C$.



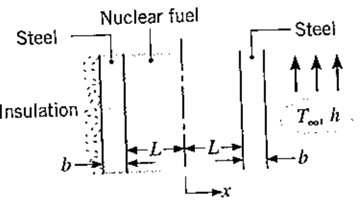
(a) Sketch the temperature distribution for case 1 on $T-x$ coordinates. Describe the key features of this distribution. Identify the location of the maximum temperature in the wall and calculate this temperature.

(b) Sketch the temperature distribution for case 2 on the same $T-x$ coordinates. Describe the key features of this distribution.

(c) What is the temperature difference between the two walls at $x = 0$ for case 2?

(d) What is the location of the maximum temperature in the composite wall of case 2? Calculate this temperature.

3.77 A nuclear fuel element of thickness $2L$ is covered with a steel cladding of thickness b. Heat generated within the nuclear fuel at a rate $\dot{q}$ is removed by a fluid at T_∞, which adjoins one surface and is characterized by a convection coefficient h. The other surface is well insulated, and the fuel and steel have thermal conductivities of k_f and k_s, respectively.



(a) Obtain an equation for the temperature distribution $T(x)$ in the nuclear fuel. Express your results in terms of $\dot{q}$, k_f, L, b, k_s, h, and T_∞.

(b) Sketch the temperature distribution $T(x)$ for the entire system.

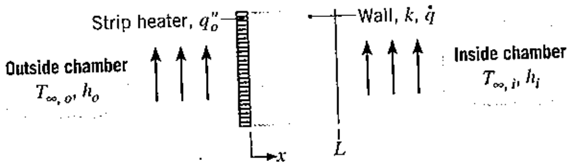
3.78 Consider the clad fuel element of Problem 3.77.

(a) Using appropriate relations from Tables C.1 and C.2, obtain an expression for the temperature distribution $T(x)$ in the fuel element. For $k_f = 60$ W/m · K, $L = 15$ mm, $b = 3$ mm, $k_s = 15$ W/m · K, $h = 10{,}000$ W/m^2 · K, and $T_\infty = 200$°C, what are the largest and smallest temperatures in the fuel element if heat is generated uniformly at a volumetric rate of $\dot{q} = 2 \times 10^7$ W/m^3? What are the corresponding locations?

(b) If the insulation is removed and equivalent convection conditions are maintained at each surface, what is the corresponding form of the temperature distribution in the fuel element? For the conditions of part (a), what are the largest and smallest temperatures in the fuel? What are the corresponding locations?

$\boxed{\text{(c)}}$ For the conditions of parts (a) and (b), plot the temperature distributions in the fuel element.

3.79 The air *inside* a chamber at $T_{\infty,i} = 50$°C is heated convectively with $h_i = 20$ W/m^2 · K by a 200-mm-thick wall having a thermal conductivity of 4 W/m · K and a uniform heat generation of 1000 W/m^3. To prevent any heat generated within the wall from being lost to the *outside* of the chamber at $T_{\infty,o} = 25$°C with $h_o = 5$ W/m^2 · K, a very thin electrical strip heater is placed on the outer wall to provide a uniform heat flux, q''_o.



(a) Sketch the temperature distribution in the wall on T–x coordinates for the condition where no heat generated within the wall is lost to the *outside* of the chamber.

(b) What are the temperatures at the wall boundaries, $T(0)$ and $T(L)$, for the conditions of part (a)?

(c) Determine the value of q''_o that must be supplied by the strip heater so that all heat generated within the wall is transferred to the *inside* of the chamber.

(d) If the heat generation in the wall were switched off while the heat flux to the strip heater remained constant, what would be the steady-state temperature, $T(0)$, of the outer wall surface?

$\boxed{\textbf{3.80}}$ In the previous problem, the strip heater acts to *guard* against heat losses from the wall to the outside, and the required heat flux q''_o depends on chamber operating conditions such as $\dot{q}$ and $T_{\infty,i}$. As a first step in designing a controller for the guard heater, compute and plot q''_o and $T(0)$ as a function of $\dot{q}$ for $200 \le \dot{q} \le 2000$ W/m^3 and $T_{\infty,i} = 30$, 50, and 70°C.

3.81 The exposed surface ($x = 0$) of a plane wall of thermal conductivity k is subjected to microwave radiation that causes volumetric heating to vary as

$$\dot{q}(x) = \dot{q}_o\left(1 - \frac{x}{L}\right)$$

where $\dot{q}_o$ (W/m^3) is a constant. The boundary at $x = L$ is perfectly insulated, while the exposed surface is maintained at a constant temperature T_o. Determine the temperature distribution $T(x)$ in terms of x, L, k, $\dot{q}_o$, and T_o.

3.82 A quartz window of thickness L serves as a viewing port in a furnace used for annealing steel. The inner surface ($x = 0$) of the window is irradiated with a uniform heat flux q''_o due to emission from hot gases in the furnace. A fraction, β, of this radiation may be assumed to be absorbed at the inner surface, while the remaining radiation is partially absorbed as it passes through the quartz. The volumetric heat generation due to this absorption may be described by an expression of the form

$$\dot{q}(x) = (1 - \beta)q''_o \alpha e^{-\alpha x}$$

where α is the absorption coefficient of the quartz. Convection heat transfer occurs from the outer surface ($x = L$) of the window to ambient air at T_∞ and is characterized by the convection coefficient h. Convection and radiation emission from the inner surface may be neglected, along with radiation emission from the outer surface. Determine the temperature distribution in the quartz, expressing your result in terms of the foregoing parameters.

3.83 For the conditions described in Problem 1.44, determine the temperature distribution, $T(r)$, in the container, expressing your result in terms of $\dot{q}_o$, r_o, T_∞, h, and the thermal conductivity k of the radioactive wastes.

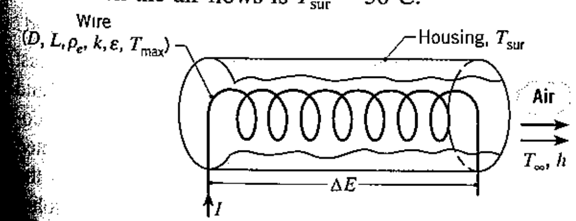
3.84 A cylindrical shell of inner and outer radii, r_i and r_o, respectively, is filled with a heat-generating material that provides a uniform volumetric generation rate (W/m^3) of $\dot{q}$. The inner surface is insulated, while the outer surface of the shell is exposed to a fluid at T_∞ and a convection coefficient h.

(a) Obtain an expression for the steady-state temperature distribution, $T(r)$, in the shell, expressing your result in terms of r_i, r_o, $\dot{q}$, h, T_∞, and the thermal conductivity k of the shell material.

(b) Determine an expression for the heat rate, $q'(r_o)$, at the outer radius of the shell in terms of $\dot{q}$ and shell dimensions.

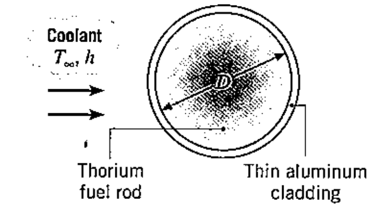
$\boxed{\textbf{3.85}}$ Consider the solid tube of Example 3.7. Using Equation (7) with Equation (10) in the *IHT* workspace, calculate and plot the temperature distributions for a tube of inner and outer radii, 50 mm and 100 mm, and a thermal conductivity of 5 W/m · K for volumetric generation rates of 1×10^5, 5×10^5, and 1×10^6 W/m^3. The inner surface is cooled by a fluid at 30°C with a convection coefficient of 1000 W/m^2 · K.

3.86 An air heater may be fabricated by coiling Nichrome wire and passing air in cross flow over the wire. Consider a heater fabricated from wire of diameter $D = 1$ mm, electrical resistivity $\rho_e = 10^{-6}$ Ω · m, thermal conductivity $k = 25$ W/m · K, and emissivity $\varepsilon = 0.20$. The heater is designed to deliver air at a temperature of $T_\infty = 50$°C under flow conditions that provide a convection coefficient of $h = 250$ W/m^2 · K for the wire. The temperature of the housing that encloses the wire and through which the air flows is $T_{sur} = 50$°C.



If the maximum allowable temperature of the wire is $T_{max} = 1200$°C, what is the maximum allowable electric current I? If the maximum available voltage is $\Delta E = 110$ V, what is the corresponding length L of wire that may be used in the heater and the power rating of the heater? *Hint*: In your solution, assume negligible temperature variations within the wire, but after obtaining the desired results, assess the validity of this assumption.

3.87 The cross section of a long cylindrical fuel element in a nuclear reactor is shown. Energy generation occurs uniformly in the thorium fuel rod, which is of diameter $D = 25$ mm and is wrapped in a thin aluminum cladding.



(a) It is proposed that, under steady-state conditions, the system operates with a generation rate of $\dot{q} = 7 \times 10^8$ W/m^3 and cooling system characteristics of $T_\infty = 95$°C and $h = 7000$ W/m^2 · K. Is this proposal satisfactory?

$\boxed{\text{(b)}}$ Explore the effect of variations in $\dot{q}$ and h by plotting temperature distributions, $T(r)$, for a range of parameter values. Suggest an envelope of acceptable operating conditions.

3.88 A nuclear reactor fuel element consists of a solid cylindrical pin of radius r_1 and thermal conductivity k_f. The fuel pin is in good contact with a cladding material of outer radius r_2 and thermal conductivity k_c. Consider steady-state conditions for which uniform heat generation occurs within the fuel at a volumetric rate $\dot{q}$ and the outer surface of the cladding is exposed to a coolant that is characterized by a temperature T_∞ and a convection coefficient h.

(a) Obtain equations for the temperature distributions $T_f(r)$ and $T_c(r)$ in the fuel and cladding, respectively. Express your results exclusively in terms of the foregoing variables.

(b) Consider a uranium oxide fuel pin for which $k_f = 2$ W/m · K and $r_1 = 6$ mm and cladding for which $k_c = 25$ W/m · K and $r_2 = 9$ mm. If $\dot{q} = 2 \times 10^8$ W/m^3, $h = 2000$ W/m^2 · K, and $T_\infty = 300$ K, what is the maximum temperature in the fuel element?

(c) Compute and plot the temperature distribution, $T(r)$, for values of h = 2000, 5000, and 10,000 $W/m^2 \cdot K$. If the operator wishes to maintain the centerline temperature of the fuel element below 1000 K, can she do so by adjusting the coolant flow and hence the value of h?

.89 Consider the configuration of Example 3.7, where uniform volumetric heating within a stainless steel tube is induced by an electric current and heat is transferred by convection to air flowing through the tube. The tube wall has inner and outer radii of r_1 = 25 mm and r_2 = 35 mm, a thermal conductivity of k = 15 $W/m \cdot K$, an electrical resistivity of $\rho_e = 0.7 \times 10^{-6}\ \Omega \cdot m$, and a maximum allowable operating temperature of 1400 K.

(a) Assuming the outer tube surface to be perfectly insulated and the air flow to be characterized by a temperature and convection coefficient of $T_{\infty,1}$ = 400 K and h_1 = 100 $W/m^2 \cdot K$, determine the maximum allowable electric current I.

(b) Compute and plot the radial temperature distribution in the tube wall for the electric current of part (a) and three values of h_1 (100, 500, and 1000 $W/m^2 \cdot K$). For each value of h_1, determine the rate of heat transfer to the air per unit length of tube.

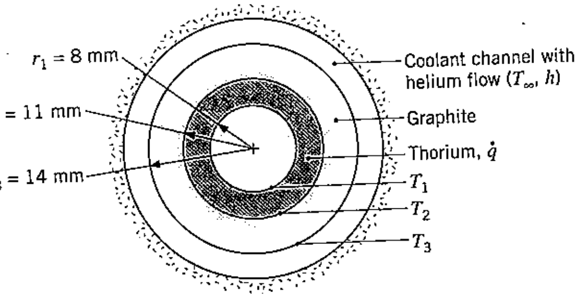
(c) In practice, even the best of insulating materials would be unable to maintain adiabatic conditions at the outer tube surface. Consider use of a refractory insulating material of thermal conductivity k = 1.0 $W/m \cdot K$ and neglect radiation exchange at its outer surface. For h_1 = 100 $W/m^2 \cdot K$ and the maximum allowable current determined in part (a), compute and plot the temperature distribution in the *composite* wall for two values of the insulation thickness (δ = 25 and 50 mm). The outer surface of the insulation is exposed to room air for which $T_{\infty,2}$ = 300 K and h_2 = 25 $W/m^2 \cdot K$. For each insulation thickness, determine the rate of heat transfer per unit tube length to the inner air flow and the ambient air.

3.90 A homeowner, whose water pipes have frozen during a period of cold weather, decides to melt the ice by passing an electric current I through the pipe wall. The inner and outer radii of the wall are designated as r_1 and r_2, and its electrical resistance per unit length is designated as R'_e (Ω/m). The pipe is well insulated on the outside, and during melting the ice (and water) in the pipe remain at a constant temperature T_m associated with the melting process.

(a) Assuming that steady-state conditions are reached shortly after application of the current, determine the form of the steady-state temperature distribution $T(r)$ in the pipe wall during the melting process.

(b) Develop an expression for the time t_m required to completely melt the ice. Calculate this time for I = 100 A, R'_e = 0.30 Ω/m, and r_1 = 50 mm.

3.91 A high-temperature, gas-cooled nuclear reactor consists of a composite cylindrical wall for which a thorium fuel element ($k \approx 57$ $W/m \cdot K$) is encased in graphite ($k \approx 3$ $W/m \cdot K$) and gaseous helium flows through an annular coolant channel. Consider conditions for which the helium temperature is T_∞ = 600 K and the convection coefficient at the outer surface of the graphite is h = 2000 $W/m^2 \cdot K$.



(a) If thermal energy is uniformly generated in the fuel element at a rate $\dot{q} = 10^8$ W/m^3, what are the temperatures T_1 and T_2 at the inner and outer surfaces, respectively, of the fuel element?

(b) Compute and plot the temperature distribution in the composite wall for selected values of $\dot{q}$. What is the maximum allowable value of $\dot{q}$?

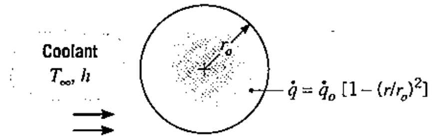
3.92 A long cylindrical rod of diameter 200 mm with thermal conductivity of 0.5 $W/m \cdot K$ experiences uniform volumetric heat generation of 24,000 W/m^3. The rod is encapsulated by a circular sleeve having an outer diameter of 400 mm and a thermal conductivity of 4 $W/m \cdot K$. The outer surface of the sleeve is exposed to cross flow of air at 27°C with a convection coefficient of 25 $W/m^2 \cdot K$.

(a) Find the temperature at the interface between the rod and sleeve and on the outer surface.

(b) What is the temperature at the center of the rod?

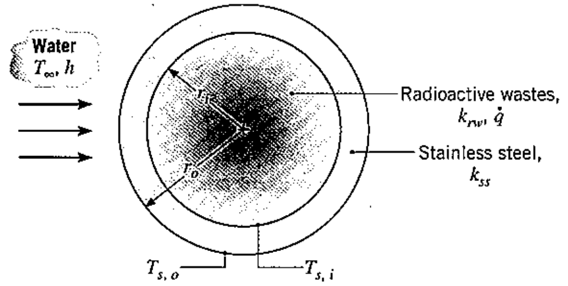
3.93 A radioactive material of thermal conductivity k is cast as a solid sphere of radius r_o and placed in a liquid bath for which the temperature T_∞ and convection coefficient h are known. Heat is uniformly generated within the solid at a volumetric rate of $\dot{q}$. Obtain the steady-state radial temperature distribution in the solid, expressing your result in terms of r_o, $\dot{q}$, k, h, and T_∞.

3.94 Radioactive wastes are packed in a thin-walled spherical container. The wastes generate thermal energy nonuniformly according to the relation $\dot{q} = \dot{q}_o[1 - (r/r_o)^2]$, where $\dot{q}$ is the local rate of energy generation per unit volume, $\dot{q}_o$ is a constant, and r_o is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at T_∞ and provides a uniform convection coefficient h.



Determine the temperature distribution, $T(r)$, in the container. Express your result in terms of $\dot{q}_o$, r_o, T_∞, h, and the thermal conductivity k of the radioactive wastes.

3.95 Radioactive wastes (k_{rw} = 20 $W/m \cdot K$) are stored in a spherical, stainless steel (k_{ss} = 15 $W/m \cdot K$) container of inner and outer radii equal to r_i = 0.5 m and r_o = 0.6 m. Heat is generated volumetrically within the wastes at a uniform rate of $\dot{q} = 10^5$ W/m^3, and the outer surface of the container is exposed to a water flow for which h = 1000 $W/m^2 \cdot K$ and T_∞ = 25°C.



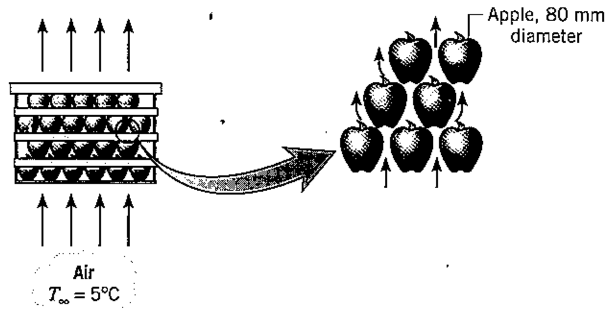
(a) Evaluate the steady-state outer surface temperature, $T_{s,o}$.

(b) Evaluate the steady-state inner surface temperature, $T_{s,i}$.

(c) Obtain an expression for the temperature distribution, $T(r)$, in the radioactive wastes. Express your result in terms of r_i, $T_{s,i}$, k_{rw}, and $\dot{q}$. Evaluate the temperature at r = 0.

(d) A proposed extension of the foregoing design involves storing waste materials having the same thermal conductivity but twice the heat generation ($\dot{q} = 2 \times 10^5$ W/m^3) in a stainless steel container of equivalent inner radius (r_i = 0.5 m). Safety considerations dictate that the maximum system temperature not exceed 475°C and that the container wall thickness be no less than t = 0.04 m and preferably at or close to the original design (t = 0.1 m). Assess the effect of varying the outside convection coefficient to a maximum achievable value of h = 5000 $W/m^2 \cdot K$ (by increasing the water velocity) and the container wall thickness. Is the proposed extension feasible? If so, recommend suitable operating and design conditions for h and t, respectively.

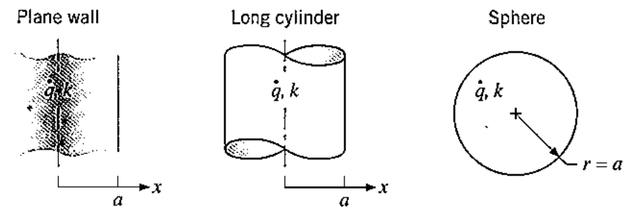
3.96 Unique characteristics of biologically active materials such as fruits, vegetables, and other products require special care in handling. Following harvest and separation from producing plants, glucose is catabolized to produce carbon dioxide, water vapor, and heat, with attendant internal energy generation. Consider a carton of apples, each of 80-mm diameter, which is ventilated with air at 5°C and a velocity of 0.5 m/s. The corresponding value of the heat transfer coefficient is 7.5 $W/m^2 \cdot K$. Within each apple thermal energy is uniformly generated at a total rate of 4000 J/kg · day. The density and thermal conductivity of the apple are 840 kg/m^3 and 0.5 $W/m \cdot K$, respectively.



(a) Determine the apple center and surface temperatures.

(b) For the stacked arrangement of apples within the crate, the convection coefficient depends on the velocity as $h = C_1 V^{0.425}$, where C_1 = 10.1 $W/m^2 \cdot K \cdot (m/s)^{0.425}$. Compute and plot the center and surface temperatures as a function of the air velocity for $0.1 \le V \le 1$ m/s.

3.97 Consider the plane wall, long cylinder, and sphere shown schematically, each with the same characteristic length a, thermal conductivity k and uniform volumetric energy generation rate $\dot{q}$.



(a) On the same graph, plot the steady-state dimensionless temperature, $[T(x \text{ or } r) - T(a)]/[(\dot{q}a^2)$

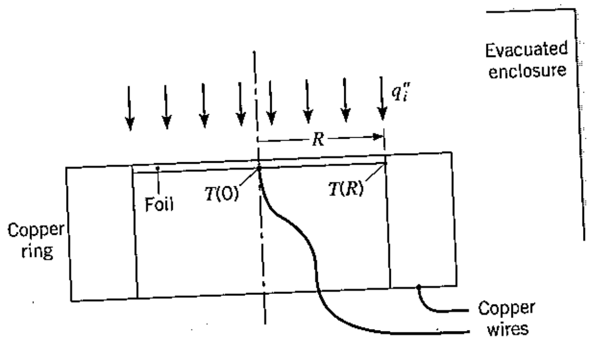
$/2k]$, versus the dimensionless characteristic length, x/a or r/a, for each shape.

(b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area.

(c) Which shape would be preferred for use as a nuclear fuel element? Explain why.

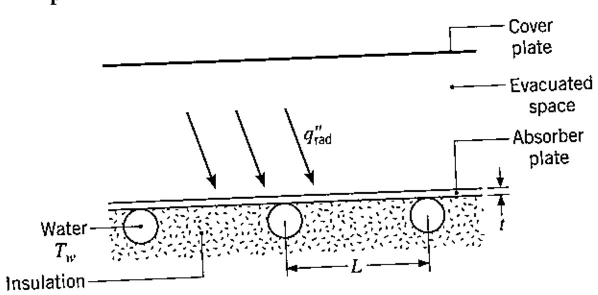
Extended Surfaces

3.98 The radiation heat gage shown in the diagram is made from constantan metal foil, which is coated black and is in the form of a circular disk of radius R and thickness t. The gage is located in an evacuated enclosure. The incident radiation flux absorbed by the foil, q_i'', diffuses toward the outer circumference and into the larger copper ring, which acts as a heat sink at the constant temperature $T(R)$. Two copper lead wires are attached to the center of the foil and to the ring to complete a thermocouple circuit that allows for measurement of the temperature difference between the foil center and the foil edge, $\Delta T = T(0) - T(R)$.



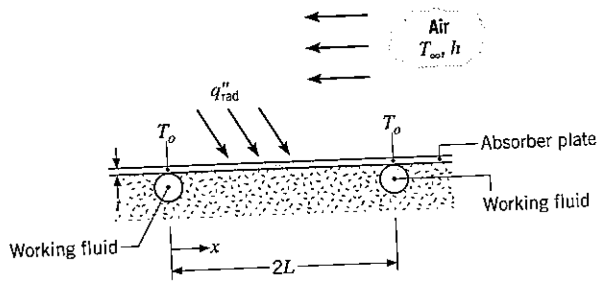
Obtain the differential equation that determines $T(r)$, the temperature distribution in the foil, under steady-state conditions. Solve this equation to obtain an expression relating ΔT to q_i''. You may neglect radiation exchange between the foil and its surroundings.

3.99 Copper tubing is joined to the absorber of a flat-plate solar collector as shown.



The aluminum alloy (2024-T6) absorber plate is 6 mm thick and well insulated on its bottom. The top surface of the plate is separated from a transparent cover plate by an evacuated space. The tubes are spaced a distance L of 0.20 m from each other, and water is circulated through the tubes to remove the collected energy. The water may be assumed to be at a uniform temperature of $T_w = 60°C$. Under steady-state operating conditions for which the *net* radiation heat flux to the surface is $q''_{rad} = 800\ W/m^2$, what is the maximum temperature on the plate and the heat transfer rate per unit length of tube? Note that q''_{rad} represents the net effect of solar radiation absorption by the absorber plate and radiation exchange between the absorber and cover plates. You may assume the temperature of the absorber plate directly above a tube to be equal to that of the water.

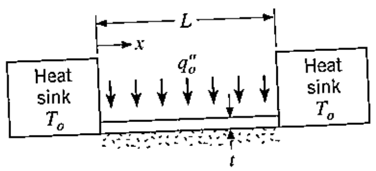
3.100 Copper tubing is joined to a solar collector plate of thickness t, and the working fluid maintains the temperature of the plate above the tubes at T_o. There is a uniform net radiation heat flux q''_{rad} to the top surface of the plate, while the bottom surface is well insulated. The top surface is also exposed to a fluid at T_∞ that provides for a uniform convection coefficient h.



(a) Derive the differential equation that governs the temperature distribution $T(x)$ in the plate.

(b) Obtain a solution to the differential equation for appropriate boundary conditions.

3.101 A thin flat plate of length L, thickness t, and width $W \gg L$ is thermally joined to two large heat sinks that are maintained at a temperature T_o. The bottom of the plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q_o''.



(a) Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.

(b) Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

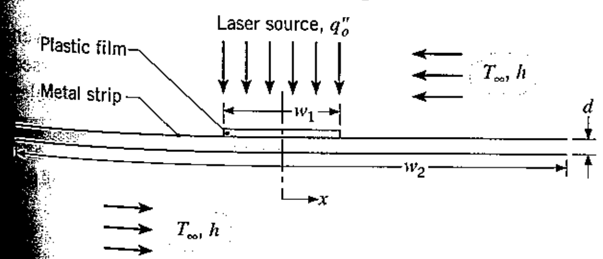
3.102 Consider the flat plate of Problem 3.101, but with the heat sinks at different temperatures, $T(0) = T_o$ and $T(L) = T_L$, and with the bottom surface no longer insulated. Convection heat transfer is now allowed to occur between this surface and a fluid at T_∞, with a convection coefficient h.

(a) Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.

(b) Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

$\boxed{(c)}$ For $q_o'' = 20{,}000\ W/m^2$, $T_o = 100°C$, $T_L = 35°C$, $T_\infty = 25°C$, $k = 25\ W/m \cdot K$, $h = 50\ W/m^2 \cdot K$, $L = 100$ mm, $t = 5$ mm, and a plate width of $W = 30$ mm, plot the temperature distribution and determine the sink heat rates, $q_x(0)$ and $q_x(L)$. On the same graph, plot three additional temperature distributions corresponding to changes in the following parameters, with the remaining parameters unchanged: (i) $q_o'' = 30{,}000\ W/m^2$, (ii) $h = 200\ W/m^2 \cdot K$, and (iii) the value of q_o'' for which $q_x(0) = 0$ when $h = 200\ W/m^2 \cdot K$.

3.103 A bonding operation utilizes a laser to provide a constant heat flux, q_o'', across the top surface of a thin adhesive-backed, plastic film to be affixed to a metal strip as shown in the sketch. The metal strip has a thickness $d = 1.25$ mm and its width is large relative to that of the film. The thermophysical properties of the strip are $\rho = 7850\ kg/m^3$, $c_p = 435\ J/kg \cdot K$, and $k = 60\ W/m \cdot K$. The thermal resistance of the plastic film of width $w_1 = 40$ mm is negligible. The upper and lower surfaces of the strip (including the plastic film) experience convection with air at 25°C and a convection coefficient of 10 $W/m^2 \cdot K$. The strip and film are very long in the direction normal to the page. Assume the edges of the metal strip are at the air temperature (T_∞).



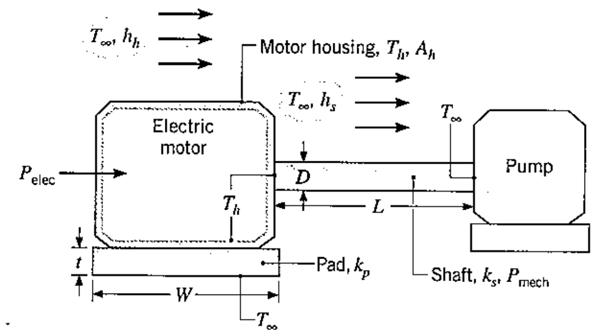
(a) Derive an expression for the temperature distribution in the portion of the steel strip with the plastic film ($-w_1/2 \le x \le +w_1/2$).

(b) If the heat flux provided by the laser is 10,000 W/m^2, determine the temperature of the plastic film at the center ($x = 0$) and its edges ($x = \pm w_1/2$).

$\boxed{(c)}$ Plot the temperature distribution for the entire strip and point out its special features.

3.104 A thin metallic wire of thermal conductivity k, diameter D, and length $2L$ is annealed by passing an electrical current through the wire to induce a uniform volumetric heat generation $\dot{q}$. The ambient air around the wire is at a temperature T_∞, while the ends of the wire at $x = \pm L$ are also maintained at T_∞. Heat transfer from the wire to the air is characterized by the convection coefficient h. Obtain an expression for the steady-state temperature distribution $T(x)$ along the wire.

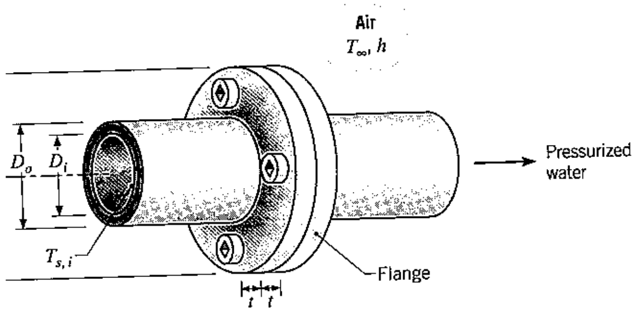
3.105 A motor draws electric power P_{elec} from a supply line and delivers mechanical power P_{mech} to a pump through a rotating copper shaft of thermal conductivity k_s, length L, and diameter D. The motor is mounted on a square pad of width W, thickness t, and thermal conductivity k_p. The surface of the housing exposed to ambient air at T_∞ is of area A_h, and the corresponding convection coefficient is h_h. Opposite ends of the shaft are at temperatures of T_h and T_∞, and heat transfer from the shaft to the ambient air is characterized by the convection coefficient h_s. The base of the pad is at T_∞.



(a) Expressing your result in terms of P_{elec}, P_{mech}, k_s, L, D, W, t, k_p, A_h, h_h, and h_s, obtain an expression for $(T_h - T_\infty)$.

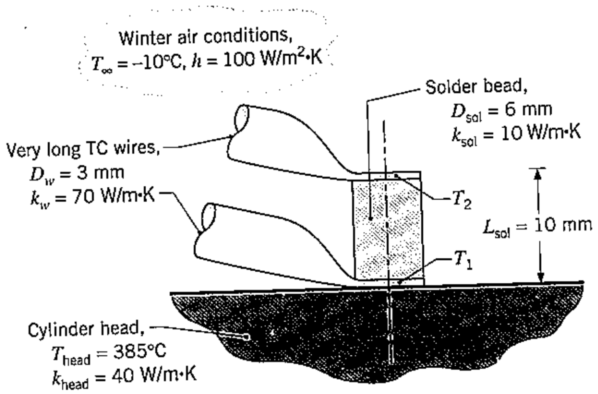
(b) What is the value of T_h if $P_{elec} = 25$ kW, $P_{mech} = 15$ kW, $k_s = 400\ W/m \cdot K$, $L = 0.5$ m, $D = 0.05$ m, $W = 0.7$ m, $t = 0.05$ m, $k_p = 0.5\ W/m \cdot K$, $A_h = 2\ m^2$, $h_h = 10\ W/m^2 \cdot K$, $h_s = 300\ W/m^2 \cdot K$, and $T_\infty = 25°C$?

3.106 Steel pipes used to transport pressurized water are bolted together by flanges of thickness $t = 15$ mm. The inner and outer diameters of the pipes are $D_i = 120$ mm and $D_o = 150$ mm, respectively, and the outer diameter of the flanges is $D_f = 250$ mm.



Under normal operating conditions, the inner surface of the pipe is at a temperature of $T_{s,i} = 300°C$, while the ambient air is at $T_\infty = 20°C$. If the convection coefficient at the flange surfaces is $h = 10$ W/m$^2\cdot$K, what is the heat loss through the flanges? The thermal conductivity of the pipe and flange is $k = 40$ W/m $\cdot$ K.

3.107 The thermocouple (TC) installation on a snowmobile engine cylinder head is shown in the schematic. The TC wire leads are attached to the upper and lower surfaces of the cylindrically shaped solder bead. The base of the bead is attached to the cylinder head operating at 385°C, and there is a thermal resistance between the bead and the cylinder head due to the *constriction* of heat flow from the head to the bead. This *constriction resistance* may be expressed as $R_{con} = 1/(2k_{head} D_{sol})$. The TC wire leads are very long and experience heat loss to the winter air at $-10°C$ with a convection coefficient of 100 W/m$^2\cdot$K. Values of other geometrical and thermal parameters are shown on the schematic.

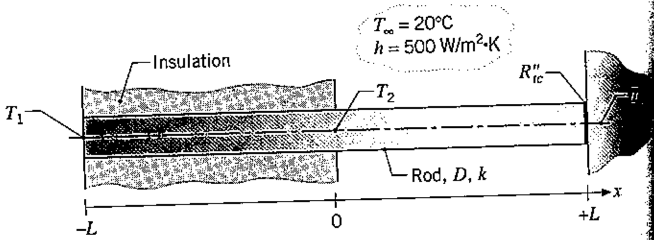


The objective of this problem is to develop a thermal model that can be used to determine the temperature difference $(T_1 - T_2)$ between the two intermediate-metal TC junctions. Assume that the solder bead does not experience any heat loss from its lateral surface.

(a) Sketch a thermal circuit of the installation, labeling temperatures, thermal resistances, and heat rates. Write expressions for each of the thermal resistances and calculate their values.

(b) Use your thermal circuit to evaluate $(T_1 - T_2)$ for the prescribed conditions. Comment on the assumptions made in building your model.

3.108 Consider a rod of diameter D, thermal conductvity k, and length $2L$ that is perfectly insulated over one portion of its length, $-L \leq x \leq 0$, and experiences convection with a fluid (T_∞, h) over the other portion, $0 \leq x \leq +L$. One end is maintained at T_1, while the other is separated from a heat sink at T_3 by an interfacial thermal contact resistance R''_{tc}.

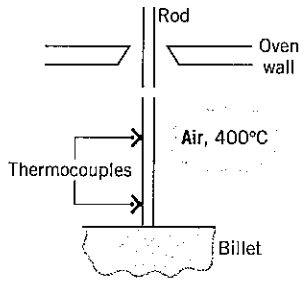


(a) Sketch the temperature distribution on $T-x$ coordinates and identify its key features. Assume that $T_1 > T_3 > T_\infty$.

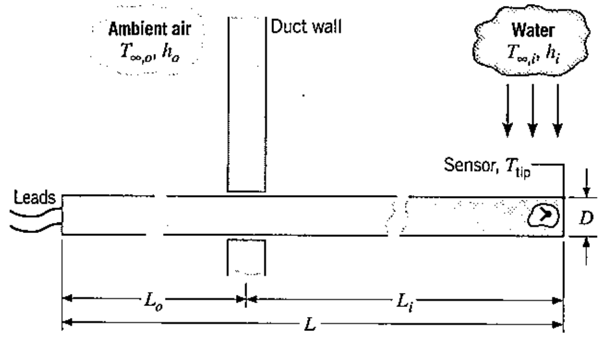
(b) Derive an expression for the midpoint temperature T_2 in terms of the thermal and geometric parameters of the system.

(c) For $T_1 = 200°C$, $T_3 = 100°C$, and the conditions provided in the schematic, calculate T_2 and plot the temperature distribution. Describe key features of the distribution and compare it to your sketch of part (a).

3.109 A long rod passes through the opening in an oven having an air temperature of 400°C and is pressed firmly onto the surface of a billet. Thermocouples imbedded in the rod at locations 25 and 120 mm from the billet register temperatures of 325 and 375°C, respectively. What is the temperature of the billet?



3.110 A probe of overall length $L = 200$ mm and diameter $D = 12.5$ mm is inserted through a duct wall such that a portion of its length, referred to as the immersion length L_i, is in contact with the water stream whose temperature, $T_{\infty,i}$, is to be determined. The convection coefficients over the immersion and ambient-exposed lengths are $h_i = 1100$ W/m$^2\cdot$K and $h_o = 10$ W/m$^2\cdot$K, respectively. The probe has a thermal conductivity of 177 W/m $\cdot$ K and is in poor thermal contact with the duct wall.

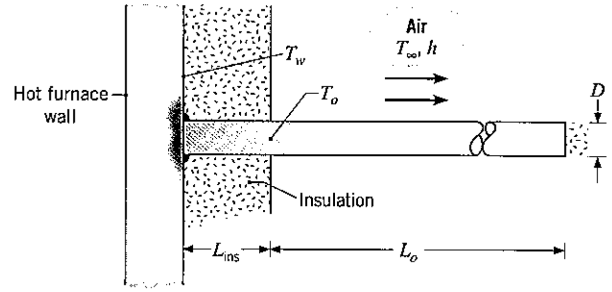


(a) Derive an expression for evaluating the measurement error, $\Delta T_{err} = T_{tip} - T_{\infty,i}$, which is the difference between the tip temperature, T_{tip}, and the water temperature, $T_{\infty,i}$. *Hint:* Define a coordinate system with the origin at the duct wall and treat the probe as two fins extending inward and outward from the duct, but having the same base temperature. Use case A results from Table 3.4.

(b) With the water and ambient air temperatures at 80 and 20°C, respectively, calculate the measurement error, ΔT_{err}, as a function of immersion length for the conditions $L_i/L = 0.225$, 0.425, and 0.625.

(c) Compute and plot the effects of probe thermal conductivity and water velocity (h_i) on the measurement error.

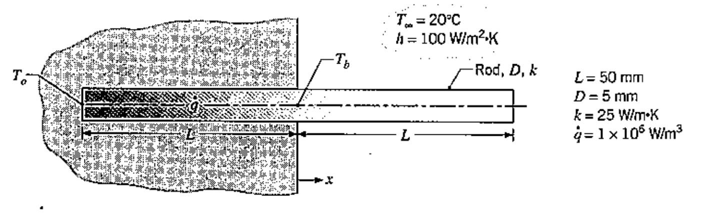
3.111 A rod of diameter $D = 25$ mm and thermal conductivity $k = 60$ W/m $\cdot$ K protrudes normally from a furnace wall that is at $T_w = 200°C$ and is covered by insulation of thickness $L_{ins} = 200$ mm. The rod is welded to the furnace wall and is used as a hanger for supporting instrumentation cables. To avoid damaging the cables, the temperature of the rod at its exposed surface, T_o, must be maintained below a specified operating limit of $T_{max} = 100°C$. The ambient air temperature is $T_\infty = 25°C$, and the convection coefficient is $h = 15$ W/m$^2\cdot$K.



(a) Derive an expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. The rod has an exposed length L_o, and its tip is well insulated.

(b) Will a rod with $L_o = 200$ mm meet the specified operating limit? If not, what design parameters would you change? Consider another material, increasing the thickness of the insulation, and increasing the rod length. Also, consider how you might attach the base of the rod to the furnace wall as a means to reduce T_o.

3.112 A metal rod of length $2L$, diameter D, and thermal conductivity k is inserted into a perfectly insulating wall, exposing one-half of its length to an air stream that is of temperature T_∞ and provides a convection coefficient h at the surface of the rod. An electromagnetic field induces volumetric energy generation at a uniform rate $\dot{q}$ within the *embedded* portion of the rod.

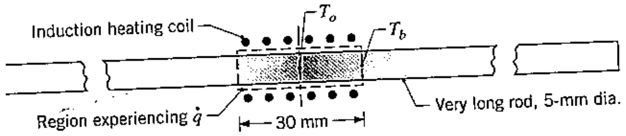


(a) Derive an expression for the steady-state temperature T_b at the base of the exposed half of the rod. The exposed region may be approximated as a very long fin.

(b) Derive an expression for the steady-state temperature T_o at the end of the embedded half of the rod.

(c) Using numerical values provided in the schematic, plot the temperature distribution in the rod and describe key features of the distribution. Does the rod behave as a very long fin?

3.113 A very long rod of 5 mm diameter and uniform thermal conductivity $k = 25$ W/m · K is subjected to a heat treatment process. The center, 30-mm-long portion of the rod within the induction heating coil experiences uniform volumetric heat generation of 7.5×10^6 W/m^3.

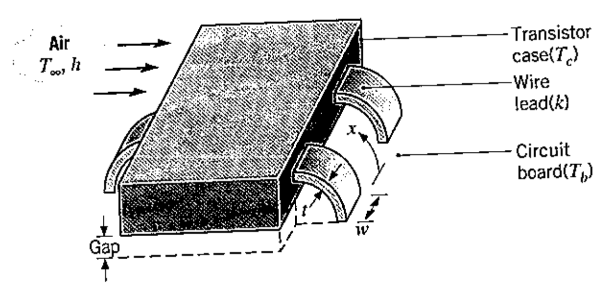


The unheated portions of the rod, which protrude from the heating coil on either side, experience convection with the ambient air at $T_\infty = 20°$C and $h = 10$ W/m^2 · K. Assume that there is no convection from the surface of the rod within the coil.

(a) Calculate the steady-state temperature T_o of the rod at the midpoint of the heated portion in the coil.

(b) Calculate the temperature of the rod T_b at the edge of the heated portion.

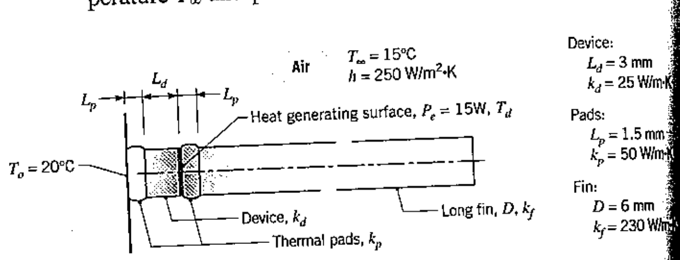
3.114 From Problem 1.71, consider the wire leads connecting the transistor to the circuit board. The leads are of thermal conductivity k, thickness t, width w, and length L. One end of a lead is maintained at a temperature T_c corresponding to the transistor case, while the other end assumes the temperature T_b of the circuit board. During steady-state operation, current flow through the leads provides for uniform volumetric heating in the amount $\dot{q}$, while there is convection cooling to air that is at T_∞ and maintains a convection coefficient h.



(a) Derive an equation from which the temperature distribution in a wire lead may be determined. List all pertinent assumptions.

(b) Determine the temperature distribution in a wire lead, expressing your results in terms of the prescribed variables.

3.115 A disk-shaped electronic device of thickness L_d, diameter D, and thermal conductivity k_d dissipates electrical power at a steady rate P_e along one of its surfaces. The device is bonded to a cooled base at T_o using a thermal pad of thickness L_p and thermal conductivity k_p. A long fin of diameter D and thermal conductivity k_f is bonded to the heat-generating surface of the device using an identical thermal pad. The fin is cooled by an air stream, which is at a temperature T_∞ and provides a convection coefficient h.

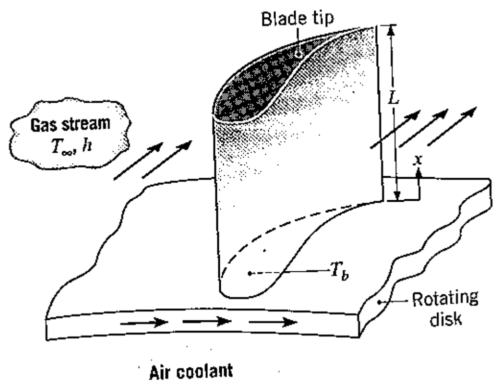


(a) Construct a thermal circuit of the system.

(b) Derive an expression for the temperature T_d of the heat-generating surface of the device in terms of the circuit thermal resistances, T_o and T_∞. Express the thermal resistances in terms of appropriate parameters.

(c) Calculate T_d for the prescribed conditions.

3.116 Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at $T_\infty = 1200°$C and maintains a convection coefficient of $h = 250$ W/m^2 · K over the blade.

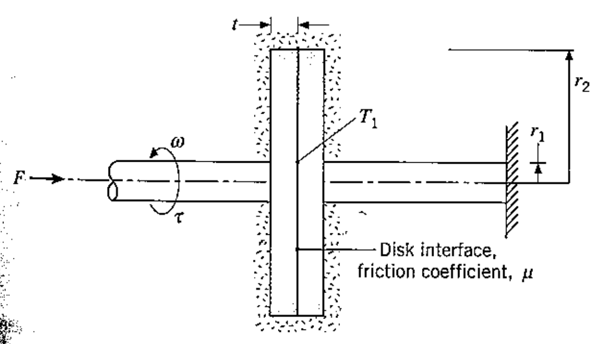


The blades, which are fabricated from Inconel, $k \approx 20$ W/m · K, have a length of $L = 50$ mm. The blade profile has a uniform cross-sectional area of $A_c = 6 \times 10^{-4}$ m^2 and a perimeter of $P = 110$ mm. A proposed blade-cooling scheme, which involves routing air through the supporting disc, is able to maintain the base of each blade at a temperature of $T_b = 300°$C.

(a) If the maximum allowable blade temperature is 1050°C and the blade tip may be assumed to be adiabatic, is the proposed cooling scheme satisfactory?

(b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?

3.117 In a test to determine the friction coefficient, μ, associated with a disc brake, one disc and its shaft are rotated at a constant angular velocity ω, while an equivalent disc/shaft assembly is stationary. Each disc has an outer radius of $r_2 = 180$ mm, a shaft radius of $r_1 = 20$ mm, a thickness of $t = 12$ mm, and a thermal conductivity of $k = 15$ W/m · K. A known force F is applied to the system, and the corresponding torque τ required to maintain rotation is measured. The disc contact pressure may be assumed to be uniform (i.e., independent of location on the interface), and the discs may be assumed to be well insulated from the surroundings.

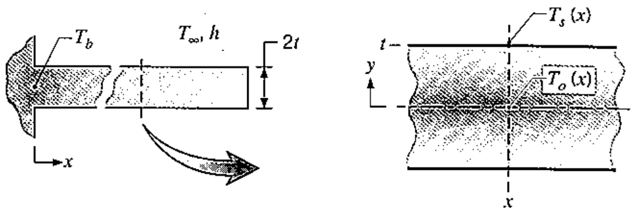


(a) Obtain an expression that may be used to evaluate μ from known quantities.

(b) For the region $r_1 \leq r \leq r_2$, determine the radial temperature distribution, $T(r)$, in the disc, where $T(r_1) = T_1$ is presumed to be known.

(c) Consider test conditions for which $F = 200$ N, $\omega = 40$ rad/s, $\tau = 8$ N · m, and $T_1 = 80°$C. Evaluate the friction coefficient and the maximum disc temperature.

3.118 Consider an extended surface of rectangular cross section with heat flow in the longitudinal direction.



In this problem we seek to determine conditions for which the transverse (y-direction) temperature gradient is negligible compared to the longitudinal (x-direction) gradient, such that the one-dimensional analysis of Section 3.6.1 is valid.

(a) Assume that the transverse temperature distribution is parabolic and of the form

$$\frac{T(y) - T_o(x)}{T_s(x) - T_o(x)} = \left(\frac{y}{t}\right)^2$$

where $T_s(x)$ is the surface temperature and $T_o(x)$ is the centerline temperature at any x-location. Using Fourier's law, write an expression for the conduction heat flux at the surface, $q''_y(t)$, in terms of T_s and T_o.

(b) Write an expression for the convection heat flux at the surface for the x-location. Equating the two expressions for the heat flux by conduction and convection, identify the parameter that determines the ratio $(T_o - T_s)/(T_s - T_\infty)$.

(c) From the foregoing analysis, develop a criterion for establishing the validity of the one-dimensional assumption used to model an extended surface.

Simple Fins

3.119 A long, circular aluminum rod is attached at one end to a heated wall and transfers heat by convection to a cold fluid.

(a) If the diameter of the rod is tripled, by how much would the rate of heat removal change?

(b) If a copper rod of the same diameter is used in place of the aluminum, by how much would the rate of heat removal change?

3.120 A brass rod 100 mm long and 5 mm in diameter extends horizontally from a casting at 200°C. The rod is in an air environment with $T_\infty = 20°$C and

$h = 30\ \text{W/m}^2 \cdot \text{K}$. What is the temperature of the rod 25, 50, and 100 mm from the casting?

3.121 The extent to which the tip condition affects the thermal performance of a fin depends on the fin geometry and thermal conductivity, as well as the convection coefficient. Consider an alloyed aluminum ($k = 180\ \text{W/m} \cdot \text{K}$) rectangular fin of length $L = 10$ mm, thickness $t = 1$ mm, and width $w \gg t$. The base temperature of the fin is $T_b = 100°\text{C}$, and the fin is exposed to a fluid of temperature $T_\infty = 25°\text{C}$.

(a) Assuming a uniform convection coefficient of $h = 100\ \text{W/m}^2 \cdot \text{K}$ over the entire fin surface, determine the fin heat transfer rate per unit width q_f', efficiency η_f, effectiveness ε_f, thermal resistance per unit width $R'_{t,f}$, and the tip temperature $T(L)$ for Cases A and B of Table 3.4. Contrast your results with those based on an *infinite fin* approximation.

$\boxed{\text{(b)}}$ Explore the effect of variations in the convection coefficient on the heat rate for $10 < h < 1000\ \text{W/m}^2 \cdot \text{K}$. Also consider the effect of such variations for a stainless steel fin ($k = 15\ \text{W/m} \cdot \text{K}$).

3.122 The extent to which the tip condition affects the thermal performance of a fin depends on the fin geometry and thermal conductivity, as well as the convection coefficient. Consider an alloyed aluminum ($k = 180\ \text{W/m} \cdot \text{K}$) rectangular fin whose base temperature is $T_b = 100°\text{C}$. The fin is exposed to a fluid of temperature $T_\infty = 25°\text{C}$, and a uniform convection coefficient of $h = 100\ \text{W/m}^2 \cdot \text{K}$ may be assumed for the fin surface.

(a) For a fin of length $L = 10$ mm, thickness $t = 1$ mm, and width $w \gg t$, determine the fin heat transfer rate per unit width q_f', efficiency η_f, effectiveness ε_f, thermal resistance per unit width $R'_{t,f}$, and tip temperature $T(L)$ for Cases A and B of Table 3.4. Contrast your results with those based on an *infinite fin* approximation.

$\boxed{\text{(b)}}$ Explore the effect of variations in L on the heat rate for $3 < L < 50$ mm. Also consider the effect of such variations for a stainless steel fin ($k = 15\ \text{W/m} \cdot \text{K}$).

3.123 A straight fin fabricated from 2024 aluminum alloy ($k = 185\ \text{W/m} \cdot \text{K}$) has a base thickness of $t = 3$ mm and a length of $L = 15$ mm. Its base temperature is $T_b = 100°\text{C}$, and it is exposed to a fluid for which $T_\infty = 20°\text{C}$ and $h = 50\ \text{W/m}^2 \cdot \text{K}$. For the foregoing conditions and a fin of unit width, compare the fin heat rate, efficiency, and volume for rectangular, triangular, and parabolic profiles.

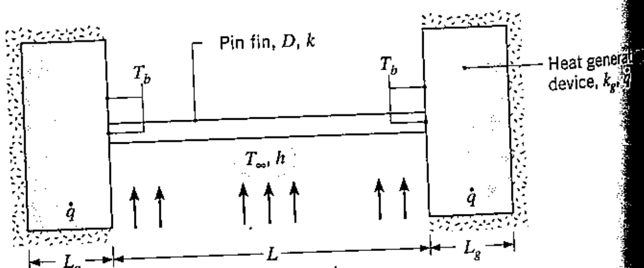
3.124 Two long copper rods of diameter $D = 10$ mm are soldered together end to end, with solder having a melting point of 650°C. The rods are in air at 25°C with a convection coefficient of $10\ \text{W/m}^2 \cdot \text{K}$. What is the minimum power input needed to effect the soldering?

3.125 Circular copper rods of diameter $D = 1$ mm and length $L = 25$ mm are used to enhance heat transfer from a surface that is maintained at $T_{s,1} = 100°\text{C}$. One end of the rod is attached to this surface (at $x = 0$), while the other end ($x = 25$ mm) is joined to a second surface, which is maintained at $T_{s,2} = 0°\text{C}$. Air flowing between the surfaces (and over the rods) is also at a temperature of $T_\infty = 0°\text{C}$, and a convection coefficient of $h = 100\ \text{W/m}^2 \cdot \text{K}$ is maintained.

(a) What is the rate of heat transfer by convection from a single copper rod to the air?

(b) What is the total rate of heat transfer from a 1-m by 1-m section of the surface at 100°C, if a bundle of the rods is installed on 4-mm centers?

3.126 Pin fins are widely used in electronic systems to provide cooling as well as to support devices. Consider the pin fin of uniform diameter D, length L, and thermal conductivity k connecting two identical devices of length L_g and surface area A_g. The devices are characterized by a uniform volumetric generation of thermal energy $\dot{q}$ and a thermal conductivity k_g. Assume that the exposed surfaces of the devices are at a uniform temperature corresponding to that of the pin base, T_b, and that heat is transferred by convection from the exposed surfaces to an adjoining fluid. The back and sides of the devices are perfectly insulated.



Derive an expression for the base temperature T_b in terms of the device parameters (k_g, $\dot{q}$, L_g, A_g), the convection parameters (T_∞, h), and the fin parameters (k, D, L).

3.127 Consider two long, slender rods of the same diameter but different materials. One end of each rod is attached to a base surface maintained at 100°C, while the surface of the rods are exposed to ambient air at 20°C. By traversing the length of each rod with a thermocouple, it was observed that the temperatures of the rods were equal at the positions $x_A = 0.15$ m and $x_B = 0.075$ m, where x is measured from the base surface. If the thermal conductivity of rod A is known to be $k_A = 70\ \text{W/m} \cdot \text{K}$, determine the value of k_B for rod B.

3.128 Consider a slender rod of length L, which is exposed to convection cooling (T_∞, h) and has both ends maintained at $T_o > T_\infty$. For each of the three cases described below, sketch the temperature distribution on T–x coordinates and identify features of the distribution. Assume the end temperatures and convection heat transfer coefficient are the same for all cases.

(a) The rod has a thermal conductivity k_A.

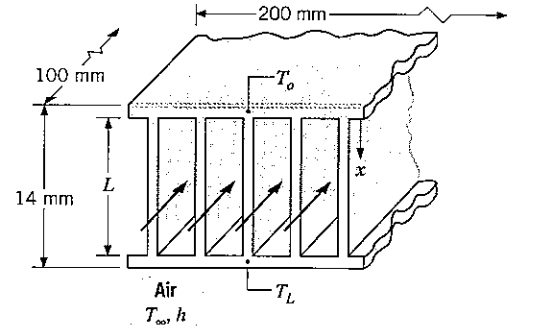
(b) The rod has a thermal conductivity k_B, where $k_B < k_A$.

(c) The rod is a composite rod with k_A for $0 \le x \le L/2$ and k_B for $L/2 \le x \le L$.

3.129 An experimental arrangement for measuring the thermal conductivity of solid materials involves the use of two long rods that are equivalent in every respect, except that one is fabricated from a standard material of known thermal conductivity k_A while the other is fabricated from the material whose thermal conductivity k_B is desired. Both rods are attached at one end to a heat source of fixed temperature T_b, are exposed to a fluid of temperature T_∞, and are instrumented with thermocouples to measure the temperature at a fixed distance x_1 from the heat source. If the standard material is aluminum, with $k_A = 200\ \text{W/m} \cdot \text{K}$, and measurements reveal values of $T_A = 75°\text{C}$ and $T_B = 60°\text{C}$ at x_1 for $T_b = 100°\text{C}$ and $T_\infty = 25°\text{C}$, what is the thermal conductivity k_B of the test material?

Fin Systems and Arrays

3.130 Finned passages are frequently formed between parallel plates to enhance convection heat transfer in compact heat exchanger cores. An important application is in electronic equipment cooling, where one or more air-cooled stacks are placed between heat-dissipating electrical components. Consider a single stack of rectangular fins of length L and thickness t, with convection conditions corresponding to h and T_∞.



(a) Obtain expressions for the fin heat transfer rates, $q_{f,o}$ and $q_{f,L}$, in terms of the base temperatures, T_o and T_L.

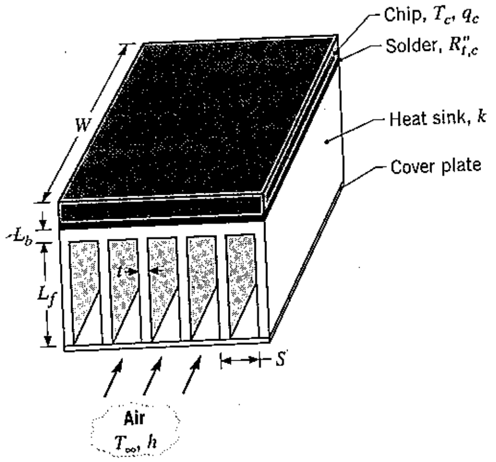
(b) In a specific application, a stack that is 200 mm wide and 100 mm deep contains 50 fins, each of length $L = 12$ mm. The entire stack is made from aluminum, which is everywhere 1.0 mm thick. If temperature limitations associated with electrical components joined to opposite plates dictate maximum allowable plate temperatures of $T_o = 400$ K and $T_L = 350$ K, what are the corresponding maximum power dissipations if $h = 150\ \text{W/m}^2 \cdot \text{K}$ and $T_\infty = 300$ K?

3.131 The fin array of Problem 3.130 is commonly found in *compact heat exchangers,* whose function is to provide a large surface area per unit volume in transferring heat from one fluid to another. Consider conditions for which the second fluid maintains equivalent temperatures at the parallel plates, $T_o = T_L$, thereby establishing symmetry about the midplane of the fin array. The heat exchanger is 1 m long in the direction of the air flow and 1 m wide in a direction normal to both the air flow and the fin surfaces. The length of the fin passages between adjoining parallel plates is $L = 8$ mm, while the fin thermal conductivity and convection coefficient are $k = 200\ \text{W/m} \cdot \text{K}$ (aluminum) and $h = 150\ \text{W/m}^2 \cdot \text{K}$, respectively.

(a) If the fin thickness and pitch are $t = 1$ mm and $S = 4$ mm, respectively, what is the value of the thermal resistance $R_{t,o}$ for a one-half section of the fin array?

$\boxed{\text{(b)}}$ Subject to the constraints that the fin thickness and pitch may not be less than 0.5 and 3 mm, respectively, assess the effect of changes in t and S.

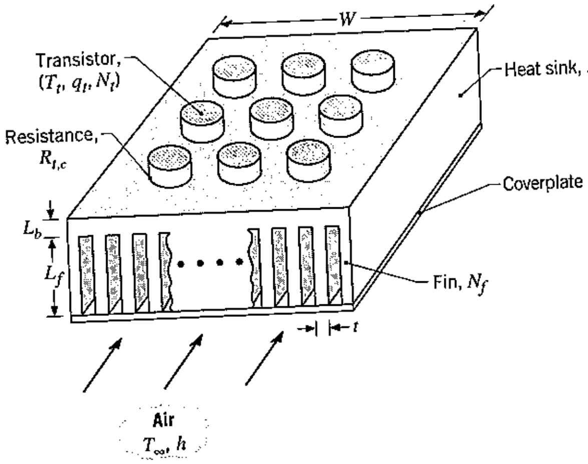
3.132 An isothermal silicon chip of width $W = 20$ mm on a side is soldered to an aluminum heat sink ($k = 180\ \text{W/m} \cdot \text{K}$) of equivalent width. The heat sink has a base thickness of $L_b = 3$ mm and an array of rectangular fins, each of length $L_f = 15$ mm. Air flow at $T_\infty = 20°C$ is maintained through channels formed by the fins and a cover plate, and for a convection coefficient of $h = 100\ W/m^2 \cdot K$, a minimum fin spacing of 1.8 mm is dictated by limitations on the flow pressure drop. The solder joint has a thermal resistance of $R''_{t,c} = 2 \times 10^{-6}\ m^2 \cdot K/W$.



(a) Consider limitations for which the array has $N = 11$ fins, in which case values of the fin thickness $t = 0.182$ mm and pitch $S = 1.982$ mm are obtained from the requirements that $W = (N - 1)S + t$ and $S - t = 1.8$ mm. If the maximum allowable chip temperature is $T_c = 85°C$, what is the corresponding value of the chip power q_c? An adiabatic fin tip condition may be assumed, and air flow along the outer surfaces of the heat sink may be assumed to provide a convection coefficient equivalent to that associated with air flow through the channels.

$\boxed{\text{(b)}}$ With $(S - t)$ and h fixed at 1.8 mm and 100 $W/m^2 \cdot K$, respectively, explore the effect of increasing the fin thickness by reducing the number of fins. With $N = 11$ and $S - t$ fixed at 1.8 mm, but relaxation of the constraint on the pressure drop, explore the effect of increasing the air flow, and hence the convection coefficient.

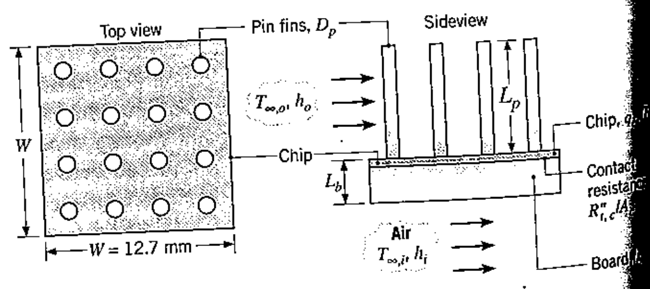
3.133 A 3 × 3 array of power transistors is attached to an aluminum heat sink ($k = 180\ W/m \cdot K$) of width $W = 150$ mm on a side. The thermal contact resistance between each transistor and the heat sink is $R_{t,c} = 0.045$ K/W. The heat sink has a base thickness of $L_b = 6$ mm and an array of $N_f = 25$ rectangular fins, each of thickness $t = 3$ mm. Cooling is provided by air flow through channels formed by the fins and a cover plate, as well as by air flow along the two sides of the heat sink (the outer surfaces of the outermost fins).



(a) Consider conditions for which the fin length is $L_f = 30$ mm, the temperature of the air is $T_\infty = 27°C$, and the convection coefficient is $h = 100\ W/m^2 \cdot K$. If the maximum allowable transistor temperature is $T_t = 100°C$, what is the maximum allowable power dissipation, q_t, per transistor? An adiabatic fin tip condition may be assumed.

$\boxed{\text{(b)}}$ Explore the effect of variations in the convection coefficient and fin length on the maximum allowable transistor power.

3.134 As more and more components are placed on a single integrated circuit (chip), the amount of heat that is dissipated continues to increase. However, this increase is limited by the maximum allowable chip operating temperature, which is approximately 75°C. To maximize heat dissipation, it is proposed that a 4 × 4 array of copper pin fins be metallurgically joined to the outer surface of a square chip that is 12.7 mm on a side.

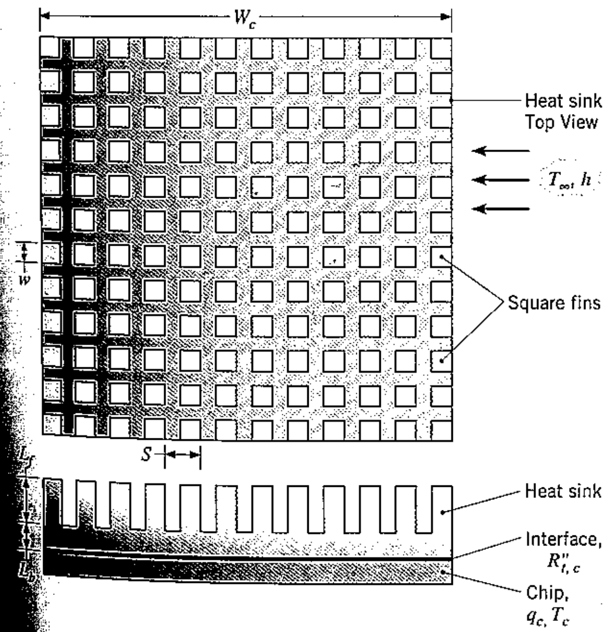


(a) Sketch the equivalent thermal circuit for the pin–chip–board assembly, assuming one-dimensional, steady-state conditions and negligible contact resistance between the pins and the chip. In variable form, label appropriate resistances, temperatures, and heat rates.

(b) For the conditions prescribed in Problem 3.27, what is the maximum rate at which heat can be dissipated in the chip when the pins are in place? That is, what is the value of q_c for $T_c = 75°C$? The pin diameter and length are $D_p = 1.5$ mm and $L_p = 15$ mm.

$\boxed{\textbf{3.135}}$ In Problem 3.134, the prescribed value of $h_o = 1000\ W/m^2 \cdot K$ is large and characteristic of liquid cooling. In practice it would be far more preferable to use air cooling, for which a reasonable upper limit to the convection coefficient would be $h_o = 250\ W/m^2 \cdot K$. Assess the effect of changes in the pin fin geometry on the chip heat rate if the remaining conditions of Problem 3.134, including a maximum allowable chip temperature of 75°C, remain in effect. Parametric variations that may be considered include the total number of pins, N, in the square array, the pin diameter D_p, and the pin length L_p. However, the product $N^{1/2}D_p$ should not exceed 9 mm to ensure adequate air flow passage through the array. Recommend a design that enhances chip cooling.

3.136 As a means of enhancing heat transfer from high-performance logic chips, it is common to attach a *heat sink* to the chip surface in order to increase the surface area available for convection heat transfer. Because of the ease with which it may be manufactured (by taking orthogonal sawcuts in a block of material), an attractive option is to use a heat sink consisting of an array of square fins of width w on a side. The spacing between adjoining fins would be determined by the width of the sawblade, with the sum of this spacing and the fin width designated as the fin pitch S. The method by which the heat sink is joined to the chip would determine the interfacial contact resistance, $R''_{t,c}$.



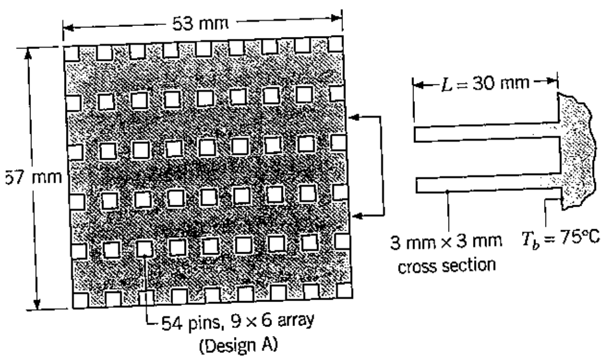
Consider a chip of width $W_c = 16$ mm and conditions for which cooling is provided by a dielectric liquid with $T_\infty = 25°C$ and $h = 1500\ W/m^2 \cdot K$. The heat sink is fabricated from copper ($k = 400\ W/m \cdot K$), and its characteristic dimensions are $w = 0.25$ mm, $S = 0.50$ mm, $L_f = 6$ mm, and $L_b = 3$ mm. The prescribed values of w and S represent minima imposed by manufacturing constraints and the need to maintain adequate flow in the passages between fins.

(a) If a metallurgical joint provides a contact resistance of $R''_{t,c} = 5 \times 10^{-6}\ m^2 \cdot K/W$ and the maximum allowable chip temperature is 85°C, what is the maximum allowable chip power dissipation q_c? Assume all of the heat to be transferred through the heat sink.

$\boxed{\text{(b)}}$ It may be possible to increase the heat dissipation by increasing w, subject to the constraint that $(S - w) \geq 0.25$ mm, and/or increasing L_f (subject to manufacturing constraints that $L_f \leq 10$ mm). Assess the effect of such changes.

$\boxed{\textbf{3.137}}$ Because of the large number of devices in today's PC chips, finned heat sinks are often used to maintain the chip at an acceptable operating temperature. Two fin designs are to be evaluated, both of which have base (unfinned) area dimensions of 53 mm × 57 mm. The fins are of square cross section and fabricated from an extruded aluminum alloy with a thermal conductivity of 175 W/m · K. Cooling air may be supplied at 25°C, and the maximum allowable chip temperature is 75°C. Other features of the design and operating conditions are tabulated.

	Fin Dimensions			
Design	Cross Section $w \times w$ (mm)	Length L (mm)	Number of Fins in Array	Convection Coefficient ($W/m^2 \cdot K$)
A	3 × 3	30	6 × 9	125
B	1 × 1	7	14 × 17	375



Determine which fin arrangement is superior. In your analysis, calculate the heat rate, efficiency, and effectiveness of a single fin, as well as the total heat rate and overall efficiency of the array. Since real estate inside the computer enclosure is important, compare the total heat rate per unit volume for the two designs.

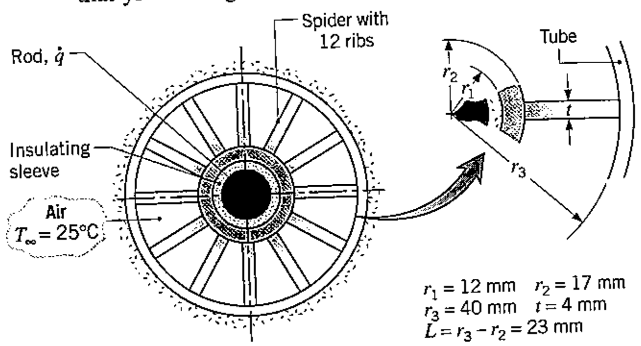
3.138 One wall of an electrical enclosure is made from copper plate ($k = 400$ W/m · K), 160 mm × 160 mm wide and 5 mm thick. To enhance heat transfer across the plate, 400 copper pin fins, each of 4-mm diameter and 20-mm length, are integrally machined on both sides of the plate in a square pitch on 8-mm centers. Warm air in the enclosure is at a temperature of 65°C, and natural circulation provides an average convection coefficient of 5 W/m² · K on the inner surface of the plate. A forced flow of ambient air at 20°C provides an average convection coefficient of 100 W/m² · K on the outer surface of the plate.

(a) Estimate the rate of heat transfer across the plate. Assuming the same convection coefficients without the fins, determine the amount of heat transfer enhancement afforded by the pins.

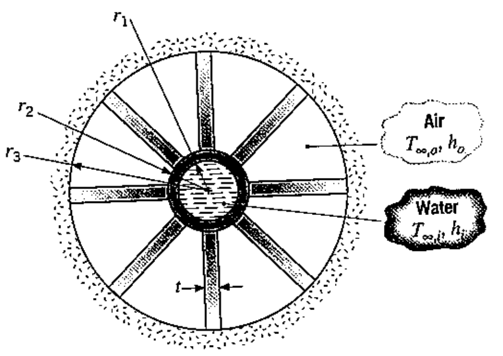
(b) It is recommended that manufacturing costs be reduced by silver soldering the pins to the plate, rather than using an expensive process such as electric discharge machining to achieve a continuous plate/pin construction. If the corresponding contact resistance is 5×10^{-6} m² · K/W, what is the rate of heat transfer across the plate?

3.139 A long rod of 20-mm diameter and a thermal conductivity of 1.5 W/m · K has a uniform internal volumetric thermal energy generation of 10^6 W/m³. The rod is covered with an electrically insulating sleeve of 2-mm thickness and thermal conductivity of 0.5 W/m · K. A spider with 12 ribs and dimensions as shown in the sketch has a thermal conductivity of 175 W/m · K, and is used to support the rod and to maintain concentricity with an 80-mm diameter tube. Air at $T_\infty = 25$°C passes over the spider surface, and the convection coefficient is 20 W/m² · K. The outer surface of the tube is well insulated.

We wish to increase volumetric heating within the rod, while not allowing its centerline temperature to exceed 100°C. Determine the impact of the following changes, which may be effected independently or concurrently: (i) increasing the air speed and hence the convection coefficient; (ii) changing the number and/or thickness of the ribs; and (iii) using an electrically nonconducting sleeve material of larger thermal conductivity (e.g., amorphous carbon or quartz). Recommend a realistic configuration that yields a significant increase in $\dot{q}$.



3.140 An air heater consists of a steel tube ($k =$ 20 W/m · K), with inner and outer radii of $r_1 =$ 13 mm and $r_2 = 16$ mm, respectively, and eight integrally machined longitudinal fins, each of thickness $t = 3$ mm. The fins extend to a concentric tube, which is of radius $r_3 = 40$ mm and insulated on its outer surface. Water at a temperature $T_{\infty,i} = 90$°C flows through the inner tube, while air at $T_{\infty,o} = 25$°C flows through the annular region formed by the larger concentric tube.



(a) Sketch the equivalent thermal circuit of the heater and relate each thermal resistance to appropriate system parameters.

(b) If $h_i = 5000$ W/m² · K and $h_o = 200$ W/m² · K, what is the heat rate per unit length?

(c) Assess the effect of increasing the number of fins N and/or the fin thickness t on the heat rate, subject to the constraint that $Nt < 50$ mm.

3.141 Determine the percentage increase in heat transfer associated with attaching aluminum fins of rectangular profile to a plane wall. The fins are 50 mm long, 0.5 mm thick, and are equally spaced at a distance of 4 mm (250 fins/m). The convection coefficient associated with the bare wall is 40 W/m² · K, while that resulting from attachment of the fins is 30 W/m² · K.

3.142 Consider the use of straight, stainless steel (304) fins of rectangular and triangular profiles on a plane wall whose temperature is 100°C. The adjoining fluid is at 20°C, and the associated convection coefficient is 75 W/m² · K. Each fin is 6 mm thick and 20 mm long. Compare the efficiency, the effectiveness, and the heat loss per unit width associated with the two kinds of fins.

3.143 Aluminum fins of triangular profile are attached to a plane wall whose surface temperature is 250°C. The fin base thickness is 2 mm, and its length is 6 mm. The system is in ambient air at a temperature of 20°C, and the surface convection coefficient is 40 W/m² · K.

(a) What are the fin efficiency and effectiveness?

(b) What is the heat dissipated per unit width by a single fin?

3.144 An annular aluminum fin of rectangular profile is attached to a circular tube having an outside diameter of 25 mm and a surface temperature of 250°C. The fin is 1 mm thick and 10 mm long, and the temperature and the convection coefficient associated with the adjoining fluid are 25°C and 25 W/m² · K, respectively.

(a) What is the heat loss per fin?

(b) If 200 such fins are spaced at 5-mm increments along the tube length, what is the heat loss per meter of tube length?

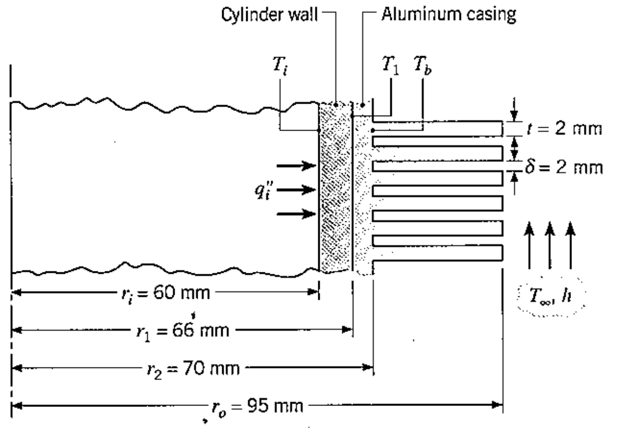
3.145 Annular aluminum fins of rectangular profile are attached to a circular tube having an outside diameter of 50 mm and an outer surface temperature of 200°C. The fins are 4 mm thick and 15 mm long. The system is in ambient air at a temperature of 20°C, and the surface convection coefficient is 40 W/m² · K.

(a) What are the fin efficiency and effectiveness?

(b) If there are 125 such fins per meter of tube length, what is the rate of heat transfer per unit length of tube?

3.146 Annular aluminum fins that are 2 mm thick and 15 mm long are installed on an aluminum tube of 30-mm diameter. The thermal contact resistance between a fin and the tube is known to be 2×10^{-4} m² · K/W. If the tube wall is at 100°C and the adjoining fluid is at 25°C, with a convection coefficient of 75 W/m² · K, what is the rate of heat transfer from a single fin? What would be the rate of heat transfer if the contact resistance could be eliminated?

3.147 It is proposed to air-cool the cylinders of a combustion chamber by joining an aluminum casing with annular fins ($k = 240$ W/m · K) to the cylinder wall ($k = 50$ W/m · K).

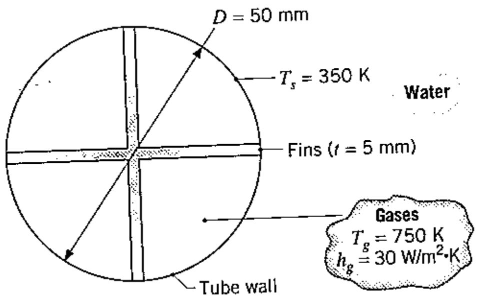


The air is at 320 K and the corresponding convection coefficient is 100 W/m² · K. Although heating at the inner surface is periodic, it is reasonable to assume steady-state conditions with a time-averaged heat flux of $q_i'' = 10^5$ W/m². Assuming negligible contact resistance between the wall and the casing, determine the wall inner temperature T_i, the interface temperature T_1, and the fin base temperature T_b. Determine these temperatures if the interface contact resistance is $R''_{t,c} = 10^{-4}$ m² · K/W.

3.148 Consider the air-cooled combustion cylinder of Problem 3.147, but instead of imposing a uniform heat flux at the inner surface, consider conditions for which the time-averaged temperature of the combustion gases is $T_g = 1100$ K and the corresponding convection coefficient is $h_g = 150$ W/m² · K. All other conditions, including the cylinder/casing contact resistance, remain the same. Determine the heat rate per unit length of cylinder (W/m), as well as the cylinder inner temperature T_i, the interface temperatures $T_{1,i}$ and $T_{1,o}$, and the fin base temperature T_b. Subject to the constraint that the fin gap is fixed at $\delta = 2$ mm, assess the effect of increasing the fin thickness at the expense of reducing the number of fins.

3.149 In Example 3.10, we considered a heat sink design and operating conditions that maintained a transistor case temperature of 80°C, while 1.63 W were dissipated by the transistor. Identify all of the measures that could be taken to improve design and/or operating conditions, such that heat dissipation may be increased while still maintaining a case temperature of 80°C. In words, assess the relative merits of each measure. Choose what you believe to be the three most promising measures, and numerically assess the effect of corresponding changes in design and/or operating conditions on thermal performance.

3.150 Water is heated by submerging 50-mm diameter, thin-walled copper tubes in a tank and passing hot combustion gases ($T_g = 750$ K) through the tubes. To enhance heat transfer to the water, four straight fins of uniform cross section, which form a cross, are inserted in each tube. The fins are 5 mm thick and are also made of copper ($k = 400$ W/m · K).



If the tube surface temperature is $T_s = 350$ K and the gas-side convection coefficient is $h_g = 30$ W/m^2 · K, what is the rate of heat transfer to the water per meter of pipe length?

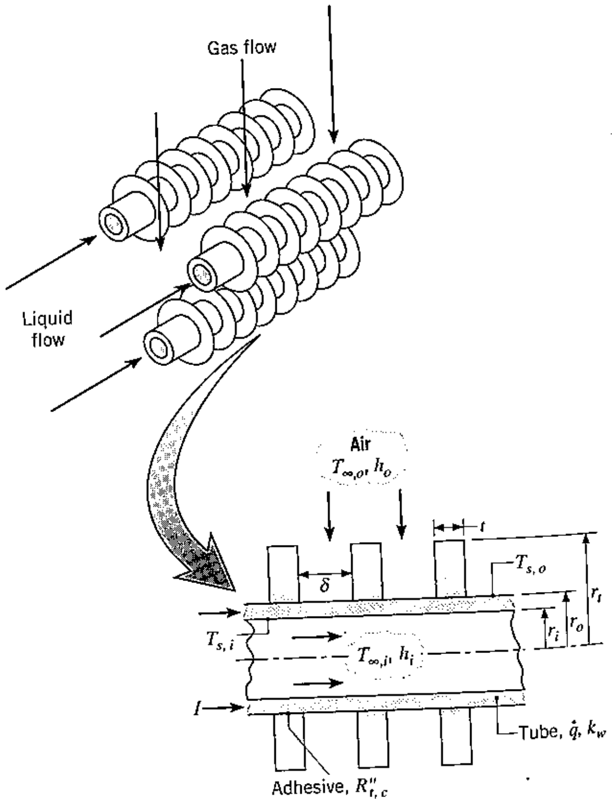
$\boxed{3.151}$ Consider the conditions of Problem 3.150, but now allow for a tube wall thickness of 5 mm (inner and outer diameters of 50 and 60 mm), a fin-to-tube thermal contact resistance of 10^{-4} m^2 · K/W, and the fact that it is the water temperature, $T_w = 350$ K, rather than the tube surface temperature, that is known. The water-side convection coefficient is $h_w = 2000$ W/m^2 · K. Determine the rate of heat transfer per unit tube length (W/m) to the water. What would be the separate effect of each of the following design changes on the heat rate: (i) elimination of the contact resistance; (ii) increasing the number of fins from four to eight; and (iii) changing the tube wall and fin material from copper to AISI 304 stainless steel ($k = 20$ W/m · K)?

3.152 A scheme for concurrently heating separate water and air streams involves passing them through and over an array of tubes, respectively, while the tube wall is heated electrically. To enhance gas-side heat transfer, annular fins of rectangular profile are attached to the outer tube surface. Attachment is facilitated with a dielectric adhesive that electrically isolates the fins from the current-carrying tube wall.

(a) Assuming uniform volumetric heat generation within the tube wall, obtain expressions for the heat rate per unit tube length (W/m) at the inner (r_i) and outer (r_o) surfaces of the wall. Express your results in terms of the tube inner and outer surface temperatures, $T_{s,i}$ and $T_{s,o}$, and other pertinent parameters.

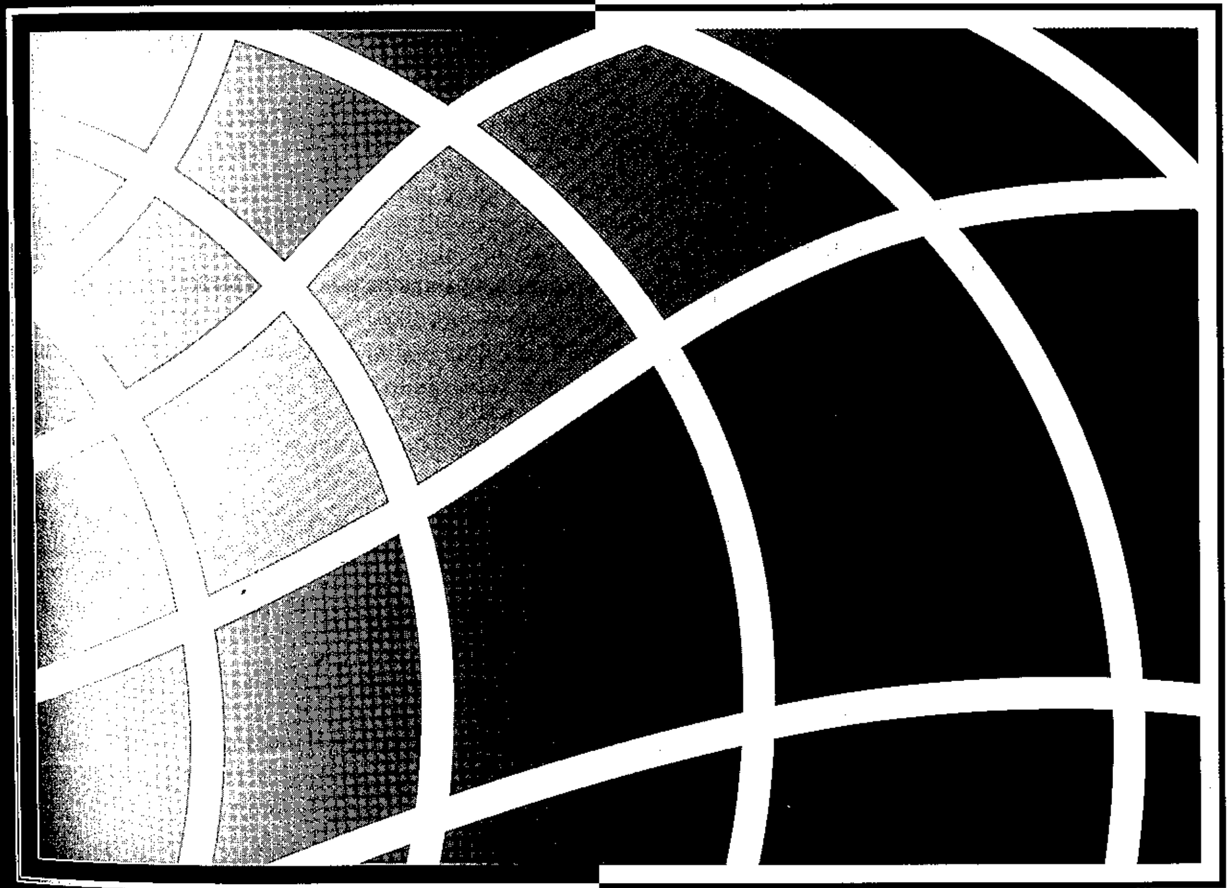
(b) Obtain expressions that could be used to determine $T_{s,i}$ and $T_{s,o}$ in terms of parameters associated with the water- and air-side conditions.

$\boxed{\text{(c)}}$ Consider conditions for which the water and air are at $T_{\infty,i} = T_{\infty,o} = 300$ K, with corresponding convection coefficients of $h_i = 2000$ W/m^2 · K and $h_o = 100$ W/m^2 · K. Heat is uniformly dissipated in a stainless steel tube ($k_w = 15$ W/m · K), having inner and outer radii of $r_i = 25$ mm and $r_o = 30$ mm, and aluminum fins ($t = \delta = 2$ mm, $r_t = 55$ mm) are attached to the outer surface, with $R''_{t,c} = 10^{-4}$ m^2 · K/W. Determine the heat rates and temperatures at the inner and outer surfaces as a function of the rate of volumetric heating $\dot{q}$. The upper limit to $\dot{q}$ will be determined by the constraints that $T_{s,i}$ not exceed the boiling point of water (100°C) and $T_{s,o}$ not exceed the decomposition temperature of the adhesive (250°C).



CHAPTER 4

Two-Dimensional, Steady-State Conduction



alloy of larger thermal conductivity and/or increasing coolant flow through the channel, thereby increasing h_i. Using the finite-difference solution with $\Delta x = \Delta y = 1$ mm, the following results are obtained for parametric variations of k and h_i:

k (W/m · K)	h_i (W/m^2 · K)	T_1 (K)	q' (W/m)
25	200	1526.0	3540.6
50	200	1523.4	3563.3
25	1000	1154.5	11,095.5
50	1000	1138.9	11,320.7

Why do increases in k and h_i reduce temperature in the blade? Why is the effect of the change in h_i more significant than that of k?

3. Note that, because the exterior surface of the blade is at an extremely high temperature, radiation losses to its surroundings may be significant. In the finite-difference analysis, such effects could be considered by linearizing the radiation rate equation (see Equations 1.8 and 1.9) and treating radiation in the same manner as convection. However, because the radiation coefficient h_r depends on the surface temperature, an iterative finite-difference solution would be necessary to ensure that the resulting surface temperatures correspond to the temperatures at which h_r is evaluated at each nodal point.
4. The *IHT* solver can be used to solve the system of 19 finite-difference equations for the nodal temperatures. The equations may be formulated by accessing the equation-builder in the *Finite-Difference Equations, 2-D, Steady-State* section of the *Tools* menu. The menu provides schematics of the control volumes (interior nodes, corner nodes, etc.), and boxes are provided to enter identifiers or subscripts for the temperatures and other parameters.
5. This example is provided as a solved model in *FEHT* and may be accessed through *Examples* on the *Toolbar*. The input screen summarizes pre- and post-processing steps, as well as the results for different nodal spacings of 1 and 0.125 mm. As an exercise, press *Run* to compute the nodal temperatures, and in the *View* menu, select *Temperature Contours* to represent the temperature field in the form of isotherms. For a tutorial on how to develop a model in *FEHT*, refer to the booklet, *Software Tools and User's Guides*, which accompanies the IHT and FEHT software.

4.6 Summary

The primary objective of this chapter was to develop an appreciation for the nature of a two-dimensional conduction problem and the methods that are available for its solution. When confronted with a two-dimensional problem, one should first determine whether an exact solution is known. This may be done by examining some of the excellent references in which exact solutions to the heat equation are obtained [1–5]. One may also want to determine whether the shape

factor is known for the system of interest [6–9]. However, often, conditions are such that the use of a shape factor or an exact solution is not possible, and it is necessary to use a finite-difference or finite-element solution. You should appreciate the inherent nature of the *discretization process* and know how to formulate and solve the finite-difference equations for the discrete points of a nodal network. You may test your understanding of related concepts by addressing the following questions.

- What is an *isotherm*? What is a *heat flow line*? How are the two lines related geometrically?
- What is an *adiabat*? How is it related to a line of symmetry? How is it intersected by an isotherm?
- What parameter characterizes the effect of geometry on the relationship between the heat rate and the overall temperature difference for steady conduction in a two-dimensional system? How is this parameter related to the conduction resistance?
- What is represented by the temperature of a *nodal point*, and how does the accuracy of a nodal temperature depend on prescription of the *nodal network*?

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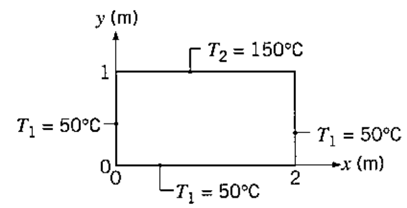
Problems

Exact Solutions

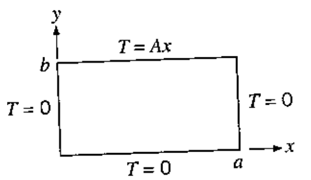
4.1 In the method of separation of variables (Section 4.2) for two-dimensional, steady-state conduction, the separation constant λ^2 in Equations 4.6 and 4.7 must be a positive constant. Show that a negative or zero value of λ^2 will result in solutions that cannot satisfy the prescribed boundary conditions.

4.2 A two-dimensional rectangular plate is subjected to prescribed boundary conditions. Using the results of the exact solution for the heat equation presented in Section 4.2, calculate the temperature at the midpoint (1, 0.5) by considering the first five nonzero terms of the infinite series that must be evaluated. Assess the error resulting from using only the first three terms of the infinite series. Plot the temperature distributions $T(x, 0.5)$ and $T(1.0, y)$.



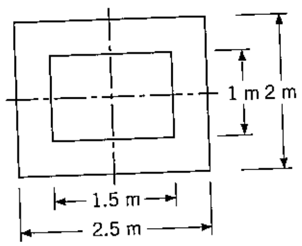
4.3 Consider the two-dimensional rectangular plate of Problem 4.2 having a thermal conductivity of 50 W/m · K. Beginning with the exact solution for the temperature distribution, derive an expression for the heat transfer rate per unit thickness from the plate along the lower surface ($0 \leq x \leq 2$, $y = 0$). Evaluate the heat rate considering the first five nonzero terms of the infinite series.

4.4 A two-dimensional rectangular plate is subjected to the boundary conditions shown. Derive an expression for the steady-state temperature distribution $T(x, y)$.

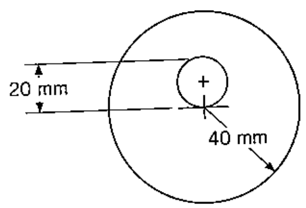


Flux Plotting

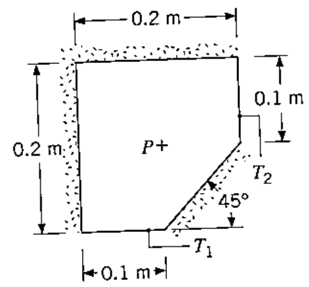
4.5 A long furnace, constructed from refractory brick with a thermal conductivity of 1.2 W/m · K, has the cross section shown with inner and outer surface temperatures of 600 and 60°C, respectively. Determine the shape factor and the heat transfer rate per unit length using the flux plot method.



4.6 A hot pipe is embedded eccentrically as shown in a material of thermal conductivity 0.5 W/m · K. Using the flux plot method, determine the shape factor and the heat transfer per unit length when the pipe and outer surface temperatures are 150 and 35°C, respectively.



4.7 A supporting strut fabricated from a material with a thermal conductivity of 75 W/m · K has the cross section shown. The end faces are at different temperatures $T_1 = 100°C$ and $T_2 = 0°C$, while the remaining sides are insulated.



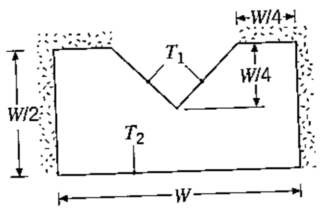
(a) Estimate the temperature at the location P.

(b) Using the flux plot method, estimate the shape factor and the heat transfer rate through the strut per unit length.

(c) Sketch the 25, 50, and 75°C isotherms.

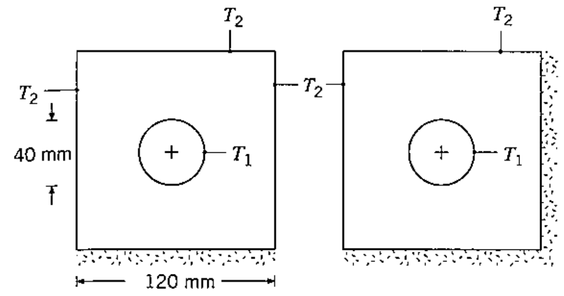
(d) Consider the same geometry, but now with the 0.1-m wide surfaces insulated, the 45° surface maintained at $T_1 = 100°C$, and the 0.2-m wide surfaces maintained at $T_2 = 0°C$. Using the flux plot method, estimate the corresponding shape factor and the heat rate per unit length. Sketch the 25, 50, and 75°C isotherms.

4.8 A hot liquid flows along a V-groove in a solid whose top and side surfaces are well insulated and whose bottom surface is in contact with a coolant.

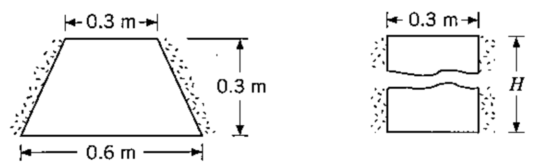


Accordingly, the V-groove surface is at a temperature T_1, which exceeds that of the bottom surface, T_2. Construct an appropriate flux plot and determine the shape factor of the system.

4.9 A very long conduit of inner circular cross section and a thermal conductivity of 1 W/m · K passes a hot fluid, which maintains the inner surface at $T_1 = 50°C$. The outer surfaces of square cross section are insulated or maintained at a uniform temperature of $T_2 = 20°C$, depending on the application. Find the shape factor and the heat rate for each case.



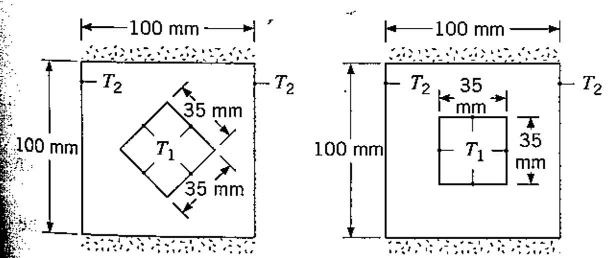
4.10 A long support column of trapezoidal cross section is well insulated on its sides, and temperatures of 100 and 0°C are maintained at its top and bottom surfaces, respectively. The column is fabricated from AISI 1010 steel, and its widths at the top and bottom surfaces are 0.3 and 0.6 m, respectively.



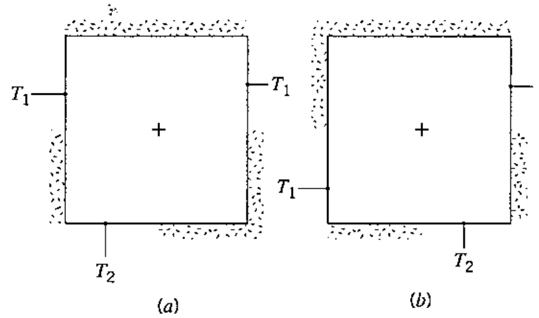
(a) Using the flux plot method, determine the heat transfer rate per unit length of the column.

(b) If the trapezoidal column is replaced by a bar of rectangular cross section 0.3 m wide and the same material, what height H must the bar be to provide an equivalent thermal resistance?

4.11 Hollow prismatic bars fabricated from plain carbon steel are 1 m long with top and bottom surfaces, as well as both ends, well insulated. For each bar, find the shape factor and the heat rate per unit length of the bar when $T_1 = 500$ K and $T_2 = 300$ K.



4.12 The two-dimensional, square shapes, 1 m to a side, are maintained at uniform temperatures, $T_1 = 100°C$ and $T_2 = 0°C$, on portions of their boundaries and are well insulated elsewhere.



Use the flux plot method to estimate the heat rate per unit length normal to the page if the thermal conductivity is 50 W/m · K.

Shape Factors

4.13 Using the thermal resistance relations developed in Chapter 3, determine shape factor expressions for the following geometries:

(a) Plane wall, cylindrical shell, and spherical shell.

(b) Isothermal sphere of diameter D buried in an infinite medium.

4.14 Radioactive wastes are temporarily stored in a spherical container, the center of which is buried a distance of 10 m below the earth's surface. The outside diameter of the container is 2 m, and 500 W of heat are released as a result of the radioactive decay process. If the soil surface temperature is 20°C, what is the outside surface temperature of the container under steady-state conditions? On a sketch of the soil–container system drawn to scale, show representative isotherms and heat flow lines in the soil.

4.15 A pipeline, used for the transport of crude oil, is buried in the earth such that its centerline is a distance of 1.5 m below the surface. The pipe has an outer diameter of 0.5 m and is insulated with a layer of cellular glass 100 mm thick. What is the heat loss per unit length of pipe under conditions for which heated oil at 120°C flows through the pipe and the surface of the earth is at a temperature of 0°C?

4.16 A long power transmission cable is buried at a depth (ground to cable centerline distance) of 2 m. The cable is encased in a thin-walled pipe of 0.1-m diameter, and to render the cable *superconducting* (essentially zero power dissipation), the space between the cable and pipe is filled with liquid nitrogen at 77 K. If the pipe is covered with a superinsulator ($k_i = 0.005$ W/m · K) of 0.05-m thickness and the surface of the earth ($k_g = 1.2$ W/m · K) is at 300 K, what is the cooling load in W/m that must be maintained by a cryogenic refrigerator per unit pipe length?

4.17 An electrical heater 100 mm long and 5 mm in diameter is inserted into a hole drilled normal to the surface of a large block of material having a thermal conductivity of 5 W/m · K. Estimate the temperature reached by the heater when dissipating 50 W with the surface of the block at a temperature of 25°C.

4.18 Two parallel pipelines spaced 0.5 m apart are buried in soil having a thermal conductivity of 0.5 W/m · K. The pipes have outer diameters of 100 and 75 mm with surface temperatures of 175 and 5°C, respectively. Estimate the heat transfer rate per unit length between the two pipelines.

4.19 A tube of diameter 50 mm having a surface temperature of 85°C is embedded in the center plane of a concrete slab 0.1 m thick with upper and lower surfaces at 20°C.

(a) Using the appropriate tabulated relation for this configuration, find the shape factor. Determine the heat transfer rate per unit lengh of the tube.

(b) Using the flux plot method, estimate the shape factor and compare with the result of part (a).

4.20 Pressurized steam at 450 K flows through a long, thin-walled pipe of 0.5-m diameter. The pipe is enclosed in a concrete casing that is of square cross section and 1.5 m on a side. The axis of the pipe is centered in the casing, and the outer surfaces of the casing are maintained at 300 K. What is the heat loss per unit length of pipe?

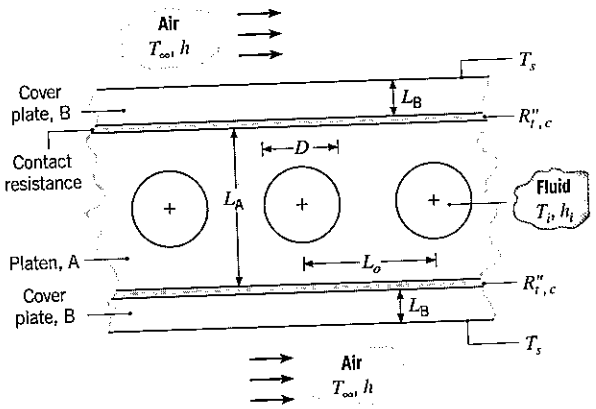
4.21 Hot water at 85°C flows through a thin-walled copper tube of 30 mm diameter. The tube is enclosed by an eccentric cylindrical shell that is maintained at 35°C and has a diameter of 120 mm. The eccentricity, defined as the separation between the centers of the tube and shell, is 20 mm. The space between the tube and shell is filled with an insulating material having a thermal conductivity of 0.05 W/m · K. Calculate the heat loss per unit length of the tube and compare the result with the heat loss for a concentric arrangement.

4.22 A furnace of cubical shape, with external dimensions of 0.35 m, is constructed from a refractory brick (fireclay). If the wall thickness is 50 mm, the inner surface temperature is 600°C, and the outer surface temperature is 75°C, calculate the heat loss from the furnace.

Shape Factors with Thermal Circuits

4.23 A cubical glass melting furnace has exterior dimensions of width $W = 5$ m on a side and is constructed from refractory brick of thickness $L = 0.35$ m and thermal conductivity $k = 1.4$ W/m · K. The sides and top of the furnace are exposed to ambient air at 25°C, with free convection characterized by an average coefficient of $h = 5$ W/m^2 · K. The bottom of the furnace rests on a framed platform for which much of the surface is exposed to the ambient air, and a convection coefficient of $h = 5$ W/m^2 · K may be assumed as a first approximation. Under operating conditions for which combustion gases maintain the inner surfaces of the furnace at 1100°C, what is the heat loss from the furnace?

4.24 A hot fluid passes through circular channels of a cast iron platen (A) of thickness $L_A = 30$ mm which is in poor contact with the cover plates (B) of thickness $L_B = 7.5$ mm. The channels are of diameter $D = 15$ mm with a centerline spacing of $L_o = 60$ mm. The thermal conductivities of the materials are $k_A = 20$ W/m · K and $k_B = 75$ W/m · K, while the contact resistance between the two materials is $R''_{t,c} = 2.0 \times 10^{-4}$ m^2 · K/W. The hot fluid is at $T_i = 150$°C, and the convection coefficient is 1000 W/m^2 · K. The cover plate is exposed to ambient air at $T_\infty = 25$°C with a convection coefficient of 200 W/m^2 · K.



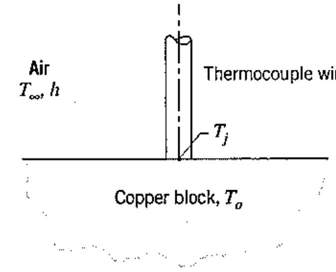
(a) Determine the heat rate from a single channel per unit length of the platen normal to the page, q'_i.

(b) Determine the outer surface temperature of the cover plate, T_s.

(c) Comment on the effects that changing the centerline spacing will have on q'_i and T_s. How would insulating the lower surface affect q'_i and T_s?

4.25 A long constantan wire of 1-mm diameter is butt welded to the surface of a large copper block, forming

a thermocouple junction. The wire behaves as a fin, permitting heat to flow from the surface, thereby depressing the sensing junction temperature T_j below that of the block T_o.

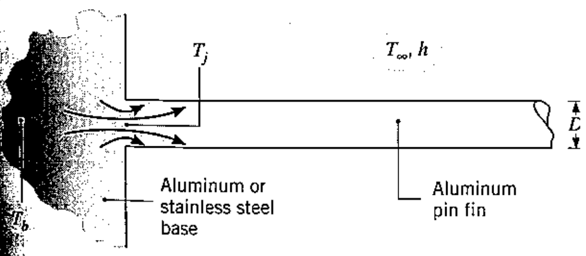


(a) If the wire is in air at 25°C with a convection coefficient of 10 W/m^2 · K, estimate the measurement error $(T_j - T_o)$ for the thermocouple when the block is at 125°C.

(b) For convection coefficients of 5, 10, and 25 W/m^2 · K, plot the measurement error as a function of the thermal conductivity of the block material over the range 15 to 400 W/m · K. Under what circumstances is it advantageous to use smaller diameter wire?

4.26 A more realistic set of conditions for Example 4.1 would involve prescription of temperatures and convection coefficients associated with fluids adjoining the inner and outer surfaces, rather than specification of the surface temperatures. Consider conditions for which the outer surfaces are exposed to ambient air, with $T_{\infty,2} = 25$°C and $h_2 = 4$ W/m^2 · K, while hot oil flowing through the hole is characterized by $T_{\infty,1} = 300$°C and $h_1 = 50$ W/m^2 · K. Determine the corresponding heat rate and surface temperatures.

4.27 In Chapter 3 we assumed that, whenever fins are attached to a base material, the base temperature is unchanged. What in fact happens is that, if the temperature of the base material exceeds the fluid temperature, attachment of a fin depresses the junction temperature T_j below the original temperature of the base, and heat flow from the base material to the fin is two-dimensional.



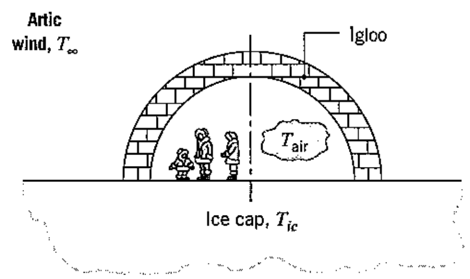
Consider conditions for which a long aluminum pin fin of diameter $D = 5$ mm is attached to a base material whose temperature far from the junction is maintained at $T_b = 100$°C. Fin convection conditions correspond to $h = 50$ W/m^2 · K and $T_\infty = 25$°C.

(a) What are the fin heat rate and junction temperature when the base material is (i) aluminum ($k = 240$ W/m · K) and (ii) stainless steel ($k = 15$ W/m · K)?

(b) Repeat the foregoing calculations if a thermal contact resistance of $R''_{t,j} = 3 \times 10^{-5}$ m^2 · K/W is associated with the method of joining the pin fin to the base material.

(c) Considering the thermal contact resistance, plot the heat rate as a function of the convection coefficient over the range $10 \leq h \leq 100$ W/m^2 · K for each of the two materials.

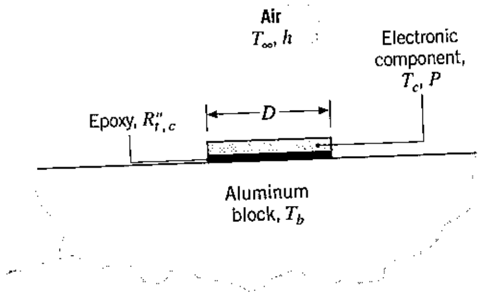
4.28 An igloo is built in the shape of a hemisphere, with an inner radius of 1.8 m and walls of compacted snow that are 0.5 m thick. On the inside of the igloo the surface heat transfer coefficient is 6 W/m^2 · K; on the outside, under normal wind conditions, it is 15 W/m^2 · K. The thermal conductivity of compacted snow is 0.15 W/m · K. The temperature of the ice cap on which the igloo sits is −20°C and has the same thermal conductivity as the compacted snow.



(a) Assuming that the occupants' body heat provides a continuous source of 320 W within the igloo, calculate the inside air temperature when the outside air temperature is $T_\infty = -40$°C. Be sure to consider heat losses through the floor of the igloo.

(b) Using the thermal circuit of part (a), perform a parameter sensitivity analysis to determine which variables have a significant effect on the inside air temperature. For instance, for very high wind conditions, the outside convection coefficient could double or even triple. Does it make sense to construct the igloo with walls half or twice as thick?

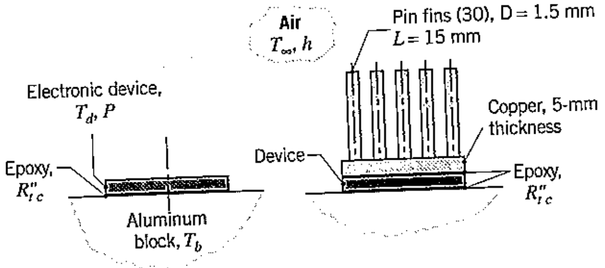
4.29 A thin, power-dissipating electronic component has a diameter of $D = 10$ mm, and one surface is epoxied to a large block of aluminum ($k = 237$ W/m · K). The thermal resistance for a unit area of the epoxy joint is $R''_{t,c} = 0.5 \times 10^{-4}$ m² · K/W, and at points well removed from the component, the block is maintained at a temperature of $T_b = 25°$C. The other surface is exposed to an airstream for which $h = 25$ W/m² · K and $T_\infty = 25°$C.



(a) Sketch the thermal circuit of the system and label thermal resistances, the heat flow direction(s), and the temperatures T_b and T_∞.

(b) If the component temperature may not exceed $T_c = 100°$C, what is the maximum allowable operating power P?

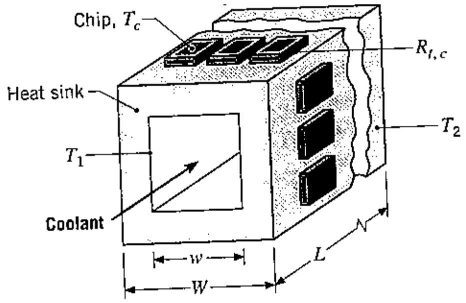
4.30 An electronic device, in the form of a disk 20 mm in diameter, dissipates 100 W when mounted flush on a large aluminum alloy (2024) block whose temperature is maintained at 27°C. The mounting arrangement is such that a contact resistance of $R''_{t,c} = 5 \times 10^{-5}$ m² · K/W exists at the interface between the device and the block.



(a) Calculate the temperature the device will reach, assuming that all the power generated by the device must be transferred by conduction to the block.

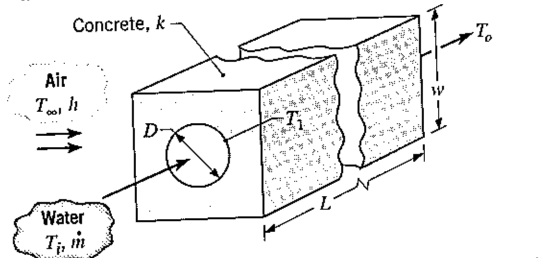
(b) In order to operate the device at a higher power level, a circuit designer proposes to attach a finned heat sink to the top of the device. The pin fins and base material are fabricated from copper ($k = 400$ W/m · K) and are exposed to an airstream at 27°C for which the convection coefficient is 1000 W/m² · K. For the device temperature computed in part (a), what is the permissible operating power?

4.31 An aluminum heat sink ($k = 240$ W/m·K) used to cool an array of electronic chips consists of a square channel of inner width $w = 25$ mm, through which liquid flow may be assumed to maintain a uniform surface temperature of $T_1 = 20°$C. The outer width and length of the channel are $W = 40$ mm and $L = 160$ mm, respectively.



If $N = 120$ chips attached to the outer surfaces of the heat sink maintain an approximately uniform surface temperature of $T_2 = 50°$C and all of the heat dissipated by the chips is assumed to be transferred to the coolant, what is the heat dissipation per chip? If the contact resistance between each chip and the heat sink is $R_{t,c} = 0.2$ K/W, what is the chip temperature?

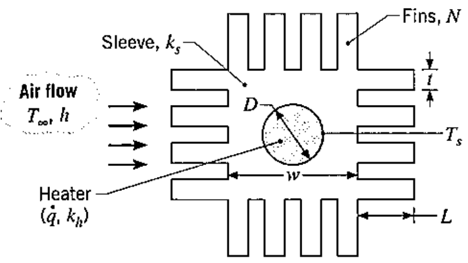
4.32 Hot water is transported from a cogeneration power station to commercial and industrial users through steel pipes of diameter $D = 150$ mm, with each pipe centered in concrete ($k = 1.4$ W/m · K) of square cross section ($w = 300$ mm). The outer surfaces of the concrete are exposed to ambient air for which $T_\infty = 0°$C and $h = 25$ W/m² · K.



(a) If the inlet temperature of water flowing through the pipe is $T_i = 90°$C, what is the heat loss per unit length of pipe in proximity to the inlet? The temperature of the pipe T_1 may be assumed to be that of the inlet water.

(b) If the difference between the inlet and outlet temperatures of water flowing through a 100-m-long pipe is not to exceed 5°C, estimate the minimum allowable flow rate $\dot{m}$. A value of $c = 4207$ J/kg · K may be used for the specific heat of the water.

4.33 The elemental unit of an air heater consists of a long circular rod of diameter D, which is encapsulated by a finned sleeve and in which thermal energy is generated by Ohmic heating. The N fins of thickness t and length L are integrally fabricated with the square sleeve of width w. Under steady-state operating conditions, the rate of thermal energy generation corresponds to the rate of heat transfer to air flow over the sleeve.



(a) Under conditions for which a uniform surface temperature T_s is maintained around the circumference of the heater and the temperature T_∞ and convection coefficient h of the air flow are known, obtain an expression for the rate of heat transfer per unit length to the air. Evaluate the heat rate for $T_s = 300°$C, $D = 20$ mm, an aluminum sleeve ($k_s = 240$ W/m · K), $w = 40$ mm, $N = 16$, $t = 4$ mm, $L = 20$ mm, $T_\infty = 50°$C, and $h = 500$ W/m² · K.

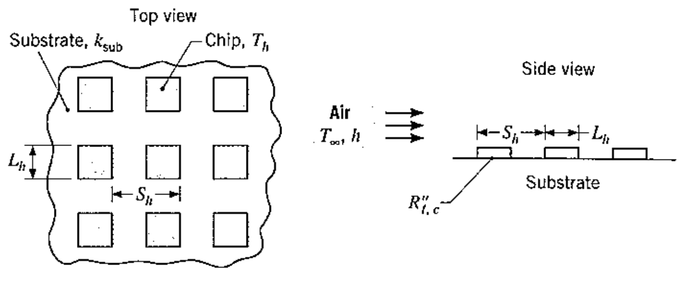
(b) For the foregoing heat rate and a copper heater of thermal conductivity $k_h = 400$ W/m·K, what is the required volumetric heat generation within the heater and its corresponding centerline temperature?

$\boxed{\text{c}}$ With all other quantities unchanged, explore the effect of variations in the fin parameters (N, L, t) on the heat rate, subject to the constraint that the fin thickness and the spacing between fins cannot be less than 2 mm.

4.34 For a small heat source attached to a large substrate, the *spreading* resistance associated with multidimensional conduction in the substrate may be approximated by the expression (Yovanovich, M. M. and V. W. Antonetti, *Adv. Thermal Modeling Elec. Comp. and Systems*, Vol. 1, A. Bar-Cohen and A. D. Kraus, Eds., Hemisphere, NY, 79–128, 1988)

$$R_{t(sp)} = \frac{1 - 1.410\,A_r + 0.344\,A_r^3 + 0.043\,A_r^5 + 0.034\,A_r^7}{4 k_{sub} A_{s,h}^{1/2}}$$

where $A_r = A_{s,h}/A_{s,sub}$ is the ratio of the heat source area to the substrate area. Consider application of the expression to an in-line array of square chips of width $L_h = 5$ mm on a side and pitch $S_h = 10$ mm. The interface between the chips and a large substrate of thermal conductivity $k_{sub} = 80$ W/m·K is characterized by a thermal contact resistance of $R''_{t,c} = 0.5 \times 10^{-4}$ m²·K/W.



If a convection heat transfer coefficient of $h = 100$ W/m²·K is associated with air flow ($T_\infty = 15°$C) over the chips and substrate, what is the maximum allowable chip power dissipation if the chip temperature is not to exceed $T_h = 85°$C?

Finite-Difference Equations: Derivations

4.35 Consider nodal configuration 2 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The horizontal boundary of the internal corner is perfectly insulated and the vertical boundary is subjected to the convection process (T_∞, h).

(b) Both boundaries of the internal corner are perfectly insulated. How does this result compare with Equation 4.45?

4.36 Consider nodal configuration 3 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The boundary is insulated. Explain how Equation 4.46 can be modified to agree with your result.

(b) The boundary is subjected to a constant heat flux.

4.37 Consider nodal configuration 4 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The upper boundary of the external corner is perfectly insulated and the side boundary is subjected to the convection process (T_∞, h).

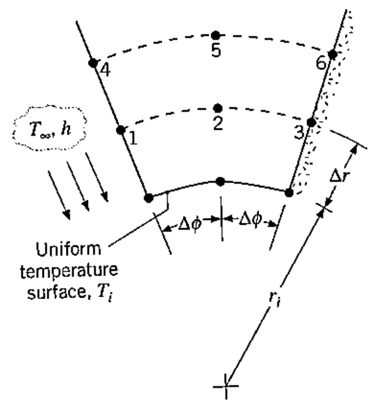
(b) Both boundaries of the external corner are perfectly insulated. How does this result compare with Equation 4.47?

4.38 Consider heat transfer in a one-dimensional (radial) cylindrical coordinate system under steady-state conditions with volumetric heat generation.

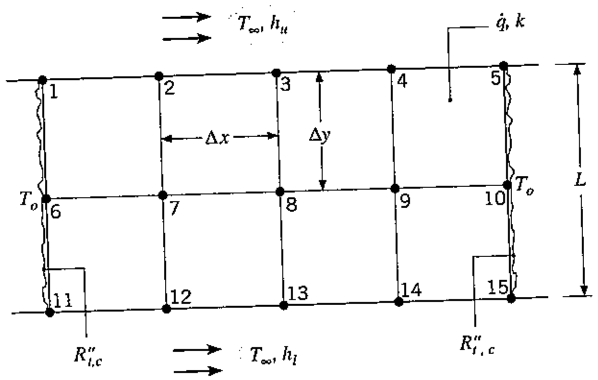
(a) Derive the finite-difference equation for any interior node m.

(b) Derive the finite-difference equation for the node n located at the external boundary subjected to the convection process (T_∞, h).

4.39 In a two-dimensional cylindrical configuration, the radial (Δr) and angular $(\Delta \phi)$ spacings of the nodes are uniform. The boundary at $r = r_i$ is of uniform temperature T_i. The boundaries in the radial direction are adiabatic (insulated) and exposed to surface convection (T_∞, h), as illustrated. Derive the finite-difference equations for (a) node 2, (b) node 3, and (c) node 1.



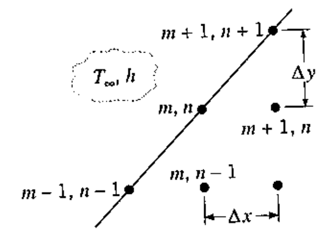
4.40 Upper and lower surfaces of a bus bar are convectively cooled by air at T_∞, with $h_u \neq h_l$. The sides are cooled by maintaining contact with heat sinks at T_o, through a thermal contact resistance of $R''_{t,c}$. The bar is of thermal conductivity k, and its width is twice its thickness L.



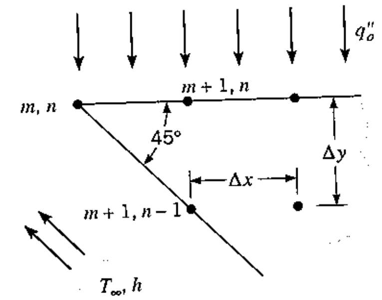
Consider steady-state conditions for which heat is uniformly generated at a volumetric rate $\dot{q}$ due to passage of an electric current. Using the energy balance method, derive finite-difference equations for nodes 1 and 13.

4.41 Derive the nodal finite-difference equations for the following configurations.

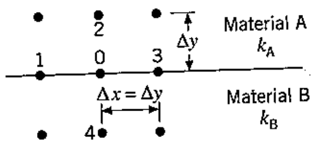
(a) Node m, n on a diagonal boundary subjected to convection with a fluid at T_∞ and a heat transfer coefficient h. Assume $\Delta x = \Delta y$.



(b) Node m, n at the tip of a cutting tool with the upper surface exposed to a constant heat flux q''_o, and the diagonal surface exposed to a convection cooling process with the fluid at T_∞ and a heat transfer coefficient h. Assume $\Delta x = \Delta y$.

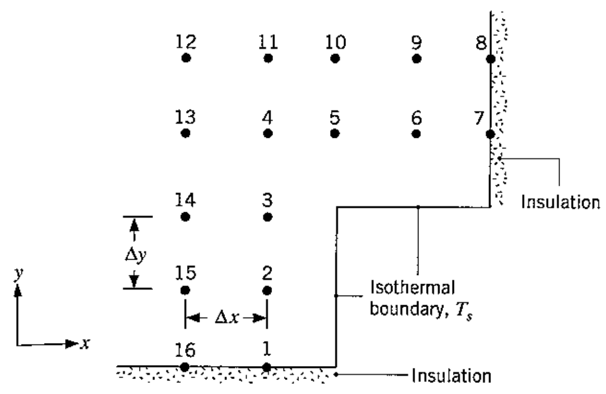


4.42 Consider the nodal point 0 located on the boundary between materials of thermal conductivity k_A and k_B.



Derive the finite-difference equation, assuming no internal generation.

4.43 Consider the two-dimensional grid $(\Delta x = \Delta y)$ representing steady-state conditions with no internal volumetric generation for a system with thermal conductivity k. One of the boundaries is maintained at a constant temperature T_s while the others are adiabatic.



Derive an expression for the heat rate per unit length normal to the page crossing the isothermal boundary (T_s).

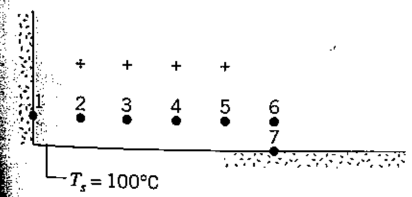
4.44 Consider a one-dimensional fin of uniform cross-sectional area, insulated at its tip, $x = L$. (See Table 3.4, case B). The temperature at the base of the fin T_b and of the adjoining fluid T_∞, as well as the heat transfer coefficient h and the thermal conductivity k, are known.

(a) Derive the finite-difference equation for any interior node m.

(b) Derive the finite-difference equation for a node n located at the insulated tip.

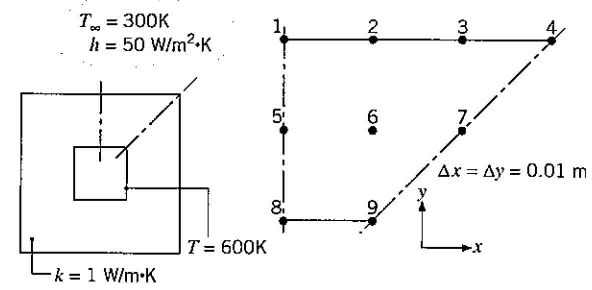
Finite-Difference Equations: Analysis

4.45 Consider the network for a two-dimensional system without internal volumetric generation having nodal temperatures shown below. If the grid space is 125 mm and the thermal conductivity of the material is 50 W/m · K, calculate the heat rate per unit length normal to the page from the isothermal surface (T_s).



Node	T_i (°C)
1	120.55
2	120.64
3	121.29
4	123.89
5	134.57
6	150.49
7	147.14

4.46 Consider the square channel shown in the sketch operating under steady-state conditions. The inner surface of the channel is at a uniform temperature of 600 K, while the outer surface is exposed to convection with a fluid at 300 K and a convection coefficient of 50 W/m² · K. From a symmetrical element of the channel, a two-dimensional grid has been constructed and the nodes labeled. The temperatures for nodes 1, 3, 6, 8, and 9 are identified.

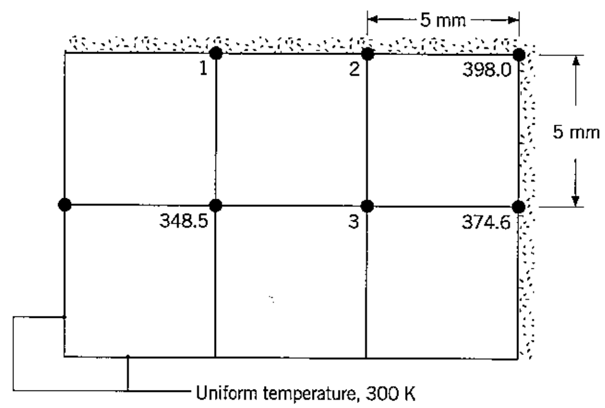


$T_1 = 430$ K $T_8 = T_9 = 600$ K
$T_3 = 394$ $T_6 = 492$

(a) Beginning with properly defined control volumes, derive the finite-difference equations for nodes 2, 4, and 7 and determine the temperatures T_2, T_4, and T_7 (K).

(b) Calculate the heat loss per unit length from the channel.

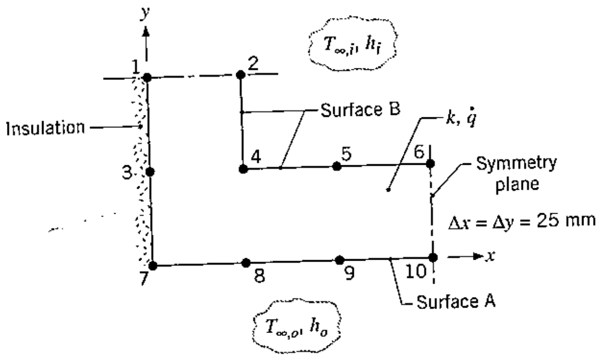
4.47 Steady-state temperatures (K) at three nodal points of a long rectangular rod are as shown. The rod experiences a uniform volumetric generation rate of 5×10^7 W/m³ and has a thermal conductivity of 20 W/m · K. Two of its sides are maintained at a constant temperature of 300 K, while the others are insulated.



(a) Determine the temperatures at nodes 1, 2, and 3.

(b) Calculate the heat transfer rate per unit length (W/m) from the rod using the nodal temperatures. Compare this result with the heat rate calculated from knowledge of the volumetric generation rate and the rod dimensions.

4.48 Steady-state temperatures at selected nodal points of the symmetrical section of a flow channel are known to be $T_2 = 95.47°C$, $T_3 = 117.3°C$, $T_5 = 79.79°C$, $T_6 = 77.29°C$, $T_8 = 87.28°C$, and $T_{10} = 77.65°C$. The wall experiences uniform volumetric heat generation of $\dot{q} = 10^6$ W/m3 and has a thermal conductivity of $k = 10\,W/m \cdot K$. The inner and outer surfaces of the channel experience convection with fluid temperatures of $T_{\infty,i} = 50°C$ and $T_{\infty,o} = 25°C$ and convection coefficients of $h_i = 500\,W/m^2 \cdot K$ and $h_o = 250\,W/m^2 \cdot K$.



(a) Determine the temperatures at nodes 1, 4, 7, and 9.

(b) Calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid.

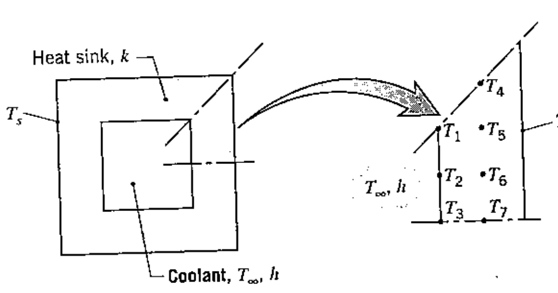
(c) Calculate the heat rate per unit length from the inner fluid to surface B.

(d) Verify that your results are consistent with an overall energy balance on the channel section.

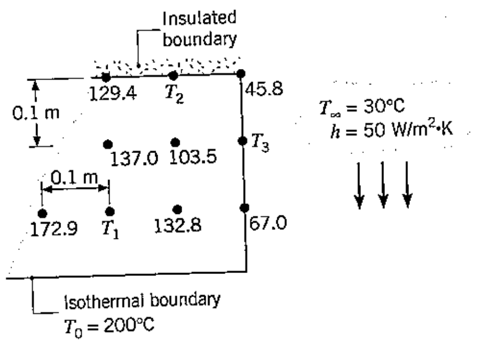
4.49 Consider an aluminum heat sink ($k = 240\,W/m \cdot K$), such as that shown schematically in Problem 4.31. The inner and outer widths of the square channel are $w = 20$ mm and $W = 40$ mm, respectively, and an outer surface temperature of $T_s = 50°C$ is maintained by the array of electronic chips. In this case, it is not the inner surface temperature that is known, but conditions (T_∞, h) associated with coolant flow through the channel, and we wish to determine the rate of heat transfer to the coolant per unit length of channel. For this purpose, consider a symmetrical section of the channel and a two-dimensional grid with $\Delta x = \Delta y = 5$ mm.

(a) For $T_\infty = 20°C$ and $h = 5000\,W/m^2 \cdot K$, determine the unknown temperatures, $T_1, \ldots, T_7$, and the rate of heat transfer per unit length of channel, q'.

$\boxed{\text{(b)}}$ Assess the effect of variations in h on the unknown temperatures and the heat rate.



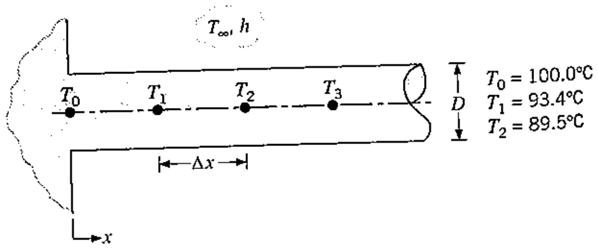
4.50 The steady-state temperatures (°C) associated with selected nodal points of a two-dimensional system having a thermal conductivity of 1.5 W/m · K are shown on the accompanying grid.



(a) Determine the temperatures at nodes 1, 2, and 3.

(b) Calculate the heat transfer rate per unit thickness normal to the page from the system to the fluid.

4.51 A steady-state, finite-difference analysis has been performed on a cylindrical fin with a diameter of 12 mm and a thermal conductivity of 15 W/m · K. The convection process is characterized by a fluid temperature of 25°C and a heat transfer coefficient of 25 W/m² · K.

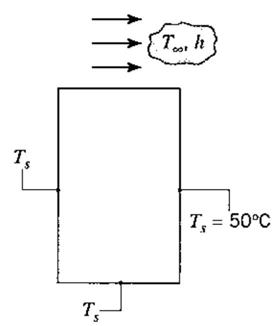


(a) The temperatures for the first three nodes, separated by a spatial increment of $x = 10$ mm, are given in the sketch. Determine the fin heat rate.

(b) Determine the temperature at node 3, T_3.

Finite-Difference Solutions

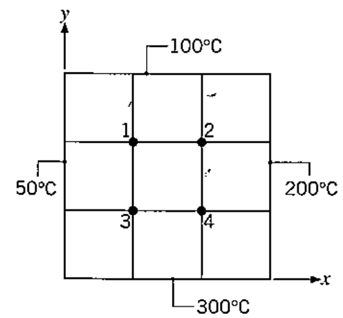
4.52 A long bar of rectangular cross section is 60 mm by 90 mm on a side and has a thermal conductivity of 1 W/m · K. One surface is exposed to a convection process with air at 100°C and a convection coefficient of 100 W/m² · K, while the remaining surfaces are maintained at 50°C.



(a) Using a grid spacing of 30 mm and the Gauss–Seidel iteration method, determine the nodal temperatures and the heat rate per unit length normal to the page into the bar from the air.

$\boxed{\text{(b)}}$ Determine the effect of grid spacing on the temperature field and heat rate. Specifically, consider a grid spacing of 15 mm. For this grid, explore the effect of changes in h on the temperature field and the isotherms.

4.53 Consider two-dimensional, steady-state conduction in a square cross section with prescribed surface temperatures.



(a) Determine the temperatures at nodes 1, 2, 3, and 4. Estimate the midpoint temperature.

$\boxed{\text{(b)}}$ Reducing the mesh size by a factor of 2, determine the corresponding nodal temperatures. Compare your results with those from the coarser grid.

$\boxed{\text{(c)}}$ From the results for the finer grid, plot the 75, 150, and 250°C isotherms.

4.54 Consider a long bar of square cross section (0.8 m to the side) and of thermal conductivity 2 W/m · K. Three of these sides are maintained at a uniform temperature of 300°C. The fourth side is exposed to a fluid at 100°C for which the convection heat transfer coefficient is 10 W/m² · K.

(a) Using an appropriate numerical technique with a grid spacing of 0.2 m, determine the midpoint temperature and heat transfer rate between the bar and the fluid per unit length of the bar.

$\boxed{\text{(b)}}$ Reducing the grid spacing by a factor of 2, determine the midpoint temperature and heat transfer rate. Plot the corresponding temperature distribution across the surface exposed to the fluid. Also, plot the 200 and 250°C isotherms.

4.55 A long conducting rod of rectangular cross section (20 mm × 30 mm) and thermal conductivity $k = 20\,W/m \cdot K$ experiences uniform heat generation at a rate $\dot{q} = 5 \times 10^7\,W/m^3$, while its surfaces are maintained at 300 K.

(a) Using a finite-difference method with a grid spacing of 5 mm, determine the temperature distribution in the rod.

$\boxed{\text{(b)}}$ With the boundary conditions unchanged, what heat generation rate will cause the midpoint temperature to reach 600 K?

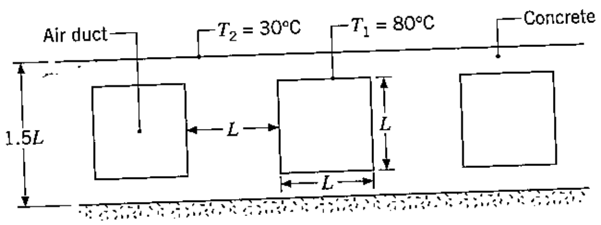
4.56 A flue passing hot exhaust gases has a square cross section, 300 mm to a side. The walls are constructed of refractory brick 150 mm thick with a thermal conductivity of 0.85 W/m · K. Calculate the heat loss from the flue per unit length when the interior and exterior surfaces are maintained at 350 and 25°C, respectively. Use a grid spacing of 75 mm.

4.57 Consider the system of Problem 4.56. The interior surface is exposed to hot gases at 350°C with a convection coefficient of 100 W/m² · K, while the exterior surface experiences convection with air at 25°C and a convection coefficient of 5 W/m² · K.

(a) Using a grid spacing of 75 mm, calculate the temperature field within the system and determine the heat loss per unit length by convection from the outer surface of the flue to the air. Compare this result with the heat gained by convection from the hot gases to the air.

$\boxed{\text{(b)}}$ Determine the effect of grid spacing on the temperature field and heat loss per unit length to the air. Specifically, consider a grid spacing of 25 mm and plot appropriately spaced isotherms on a schematic of the system. Explore the effect of changes in the convection coefficients on the temperature field, and heat loss.

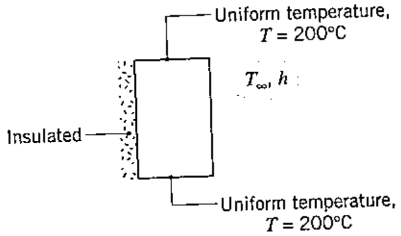
4.58 A common arrangement for heating a large surface area is to move warm air through rectangular ducts below the surface. The ducts are square and located midway between the top and bottom surfaces that are exposed to room air and insulated, respectively.



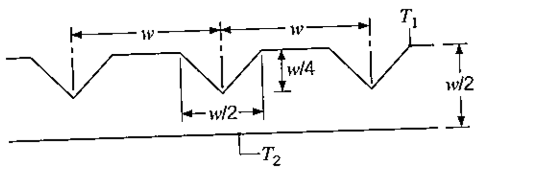
For the condition when the floor and duct temperatures are 30 and 80°C, respectively, and the thermal conductivity of concrete is 1.4 W/m · K, calculate the heat rate from each duct, per unit length of duct. Use a grid spacing with $\Delta x = 2\,\Delta y$, where $\Delta y = 0.125L$ and $L = 150$ mm.

$\boxed{\text{4.59}}$ Consider the gas turbine cooling scheme of Example 4.4. In Problem 3.23, advantages associated with applying a *thermal barrier coating* (TBC) to the exterior surface of a turbine blade are described. If a 0.5-mm-thick zirconia coating (k=1.3 W/m · K, $R''_{t,c} = 10^{-4}$ m² · K/W) is applied to the outer surface of the air-cooled blade, determine the temperature field in the blade for the operating conditions of Example 4.4.

4.60 A long bar of rectangular cross section, 0.4 m × 0.6 m on a side and having a thermal conductivity of 1.5 W/m · K, is subjected to the boundary conditions shown below. Two of the sides are maintained at a uniform temperature of 200°C. One of the sides is adiabatic, and the remaining side is subjected to a convection process with T_∞ = 30°C and h = 50 W/m² · K. Using an appropriate numerical technique with a grid spacing of 0.1 m, determine the temperature distribution in the bar and the heat transfer rate between the bar and the fluid per unit length of the bar.



4.61 The top surface of a plate, including its grooves, is maintained at a uniform temperature of T_1 = 200°C. The lower surface is at T_2 = 20°C, the thermal conductivity is 15 W/m · K, and the groove spacing is 0.16 m.

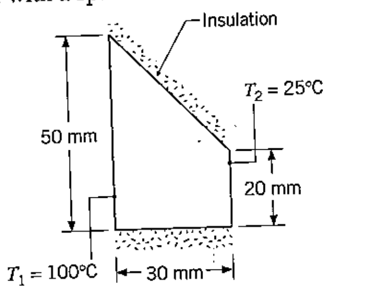


(a) Using a finite-difference method with a mesh size of $\Delta x = \Delta y$ = 40 mm, calculate the unknown nodal temperatures and the heat transfer rate per width of groove spacing (w) and per unit length normal to the page.

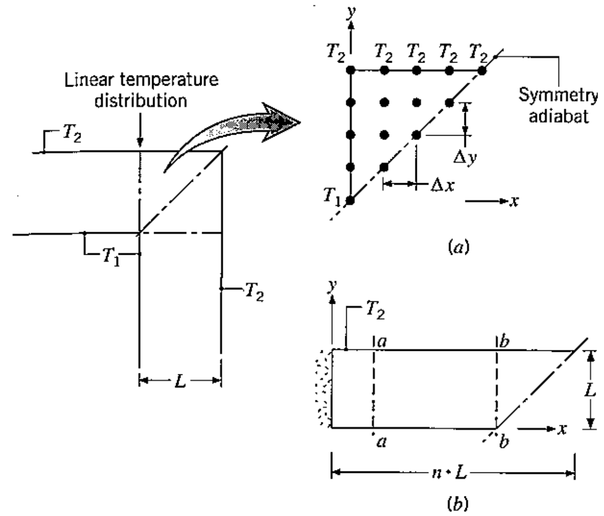
$\boxed{\text{(b)}}$ With a mesh size of $\Delta x = \Delta y$ = 10 mm, repeat the foregoing calculations, determining the temperature field and the heat rate. Also, consider conditions for which the bottom surface is not at a uniform temperature T_2 but is exposed to a fluid at T_∞ = 20°C. With $\Delta x = \Delta y$ = 10 mm, determine the temperature field and heat rate for values of h = 5, 200, and 1000 W/m² · K, as well as for $h \rightarrow \infty$.

4.62 Refer to the two-dimensional rectangular plate of Problem 4.2. Using an appropriate numerical method with $\Delta x = \Delta y$ = 0.25 m, determine the temperature at the midpoint (1, 0.5).

4.63 A long trapezoidal bar is subjected to uniform temperatures on two surfaces, while the remaining surfaces are well insulated. If the thermal conductivity of the material is 20 W/m · K, estimate the heat transfer rate per unit length of the bar using a finite-difference method. Use the Gauss–Seidel method of solution with a space increment of 10 mm.



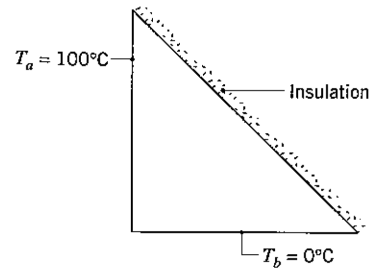
$\boxed{\text{4.64}}$ The shape factor for conduction through the edge of adjoining walls for which $D > L/5$, where D and L are the wall depth and thickness, respectively, is shown in Table 4.1. The two-dimensional symmetrical element of the edge, which is represented by inset (a), is bounded by the diagonal symmetry adiabat and a section of the wall thickness over which the temperature distribution is assumed to be linear between T_1 and T_2.



(a) Using the nodal network of inset (a) with L = 40 mm, determine the temperature distribution in the element for T_1 = 100°C and T_2 = 0°C. Evaluate the heat rate per unit depth (D = 1 m) if k = 1 W/m · K. Determine the corresponding shape factor for the edge and compare your result with that from Table 4.1.

(b) Choosing a value of n = 1 or n = 1.5, establish a nodal network for the trapezoid of inset (b) and determine the corresponding temperature field. Assess the validity of assuming linear temperature distributions across sections a–a and b–b.

$\boxed{\text{4.65}}$ The diagonal of a long triangular bar is well insulated, while sides of equivalent length are maintained at uniform temperatures T_a and T_b.

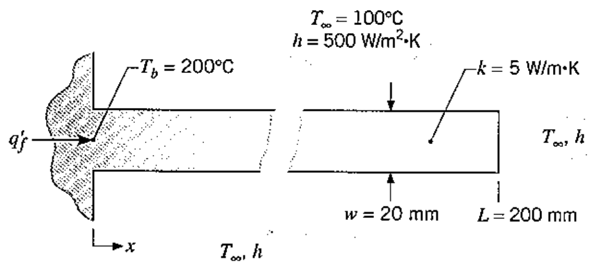


(a) Establish a nodal network consisting of five nodes along each of the sides. For one of the nodes on the diagonal surface, define a suitable control volume and derive the corresponding finite-difference equation. Using this form for the diagonal nodes and appropriate equations for the interior nodes, find the temperature distribution for the bar. On a scale drawing of the shape, show the 25, 50, and 75°C isotherms.

(b) An alternate and simpler procedure to obtain the finite-difference equations for the diagonal nodes follows from recognizing that the insulated diagonal surface is a symmetry plane. Consider a square 5 × 5 nodal network and represent its diagonal as a symmetry line. Recognize which nodes on either side of the diagonal have identical temperatures. If you have done this properly, you can treat the diagonal nodes as "interior" nodes and write the finite-difference equations by inspection.

Finite-Element Solutions

$\boxed{\text{4.66}}$ A straight fin of uniform cross section is fabricated from a material of thermal conductivity k = 5 W/m · K, thickness w = 20 mm, and length L = 200 mm. The fin is very long in the direction normal to the page. The base of the fin is maintained at T_b = 200°C, and the tip condition allows for convection (Case A, if Table 3.4), with h = 500 W/m² · K and T_∞ = 25°C.



(a) Assuming one-dimensional heat transfer in the fin, calculate the fin heat rate, q'_f (W/m), and the tip temperature T_L. Calculate the Biot number for the fin to determine whether the one-dimensional assumption is valid.

(b) Using the finite-element method of *FEHT*, perform a two-dimensional analysis on the fin to determine the fin heat rate and tip temperature. Compare your results with those from the one-dimensional, analytical solution of Part (a). Use the *View/Temperature Contours* option to display isotherms, and discuss key features of the corresponding temperature field and heat flow pattern. *Hint*: In drawing the outline of the fin, take advantage of symmetry. Use a fine mesh near the base and a coarser mesh near the tip. Why?

(c) Validate your *FEHT* model by comparing predictions with the analytical solution for a fin with thermal conductivities of $k = 50\,\mathrm{W/m \cdot K}$ and $500\,\mathrm{W/m \cdot K}$. Is the one-dimensional heat transfer assumption valid for these conditions?

4.67 Consider the long rectangular bar of Problem 4.52 with the prescribed boundary conditions.

(a) Using the finite-element method of *FEHT*, determine the temperature distribution. Use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the distribution.

(b) Using the *View/Heat Flows* command, calculate the heat rate per unit length (W/m) from the bar to the air stream.

(c) Explore the effect on the heat rate of increasing the convection coefficient by factors of two and three. Explain why the change in the heat rate is not proportional to the change in the convection coefficient.

4.68 Consider the long rectangular rod of Problem 4.55, which experiences uniform heat generation while its surfaces are maintained at a fixed temperature.

(a) Using the finite-element method of *FEHT*, determine the temperature distribution. Use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the distribution.

(b) With the boundary conditions unchanged, what heat generation rate will cause the midpoint temperature to reach 600 K?

4.69 Consider the symmetrical section of the flow channel of Problem 4.48, with the prescribed values of $\dot{q}$, $k, T_{\infty,i}, T_{\infty,o}, h_i$ and h_o. Use the finite-element method of *FEHT* to obtain the following results.

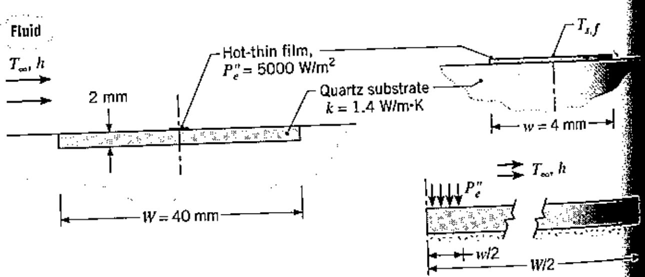
(a) Determine the temperature distribution in the symmetrical section, and use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the temperature distribution, including the hottest and coolest regions and the region with the steepest gradients. Describe the heat flow field.

(b) Using the *View/Heat Flows* command, calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid.

(c) Calculate the heat rate per unit length from the inner fluid to surface B.

(d) Verify that your results are consistent with an overall energy balance on the channel section.

4.70 The hot-film heat flux gage shown schematically may be used to determine the convection coefficient of an adjoining fluid stream by measuring the electric power dissipation per unit area, $P_e''(\mathrm{W/m^2})$, and the average surface temperature, $T_{s,f}$, of the film. The power dissipated in the film is transferred directly to the fluid by convection, as well as by conduction into the substrate. If substrate conduction is negligible, the gage measurements can be used to determine the convection coefficient without application of a correction factor. Your assignment is to perform a two-dimensional, steady-state conduction analysis to estimate the fraction of the power dissipation that is conducted into a 2-mm-thick quartz substrate of width $W = 40\,\mathrm{mm}$ and thermal conductivity $k = 1.4\,\mathrm{W/m \cdot K}$. The thin, hot-film gage has a width of $w = 4\,\mathrm{mm}$ and operates at a uniform power dissipation of $5000\,\mathrm{W/m^2}$. Consider cases for which the fluid temperature is 25°C and the convection coefficient is 500, 1000, and $2000\,\mathrm{W/m^2 \cdot K}$.

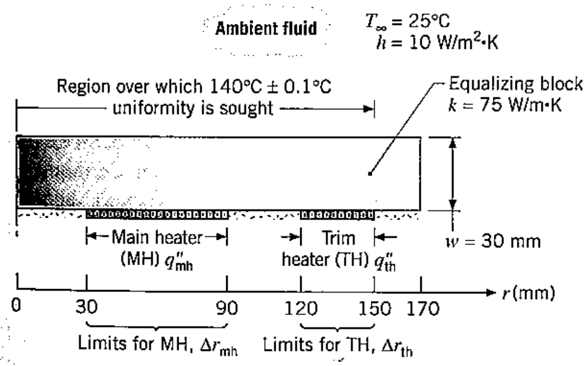


Use the finite-element method of *FEHT* to analyze a symmetrical half-section of the gage and the quartz substrate. Assume that the lower and end surfaces of the substrate are perfectly insulated, while the upper surface experiences convection with the fluid.

(a) Determine the temperature distribution and the conduction heat rate into the region below the hot film for the three values of h. Calculate the fractions of electric power dissipation represented by these rates. *Hint*: Use the *View/Heat Flow* command to find the heat rate across the boundary elements.

(b) Use the *View/Temperature Contours* command to view the isotherms and heat flow patterns. Describe the heat flow paths, and comment on features of the gage design that influence the paths. What limitations on applicability of the gage have been revealed by your analysis?

4.71 A semiconductor industry roadmap for microlithography processing requires that a 300-mm-diameter silicon wafer be maintained at a steady-state temperature of 140°C to within a uniformity of 0.1°C. The design of a hot-plate tool to hopefully meet this requirement is shown schematically. An *equalizing block* (EB), on which the wafer would be placed, is fabricated from an aluminum alloy of thermal conductivity $k = 75\,\mathrm{W/m \cdot K}$ and is heated by two ring-shaped electrical heaters. The two-zone heating arrangement allows for independent control of a main heater (MH) and a trim heater (TH), which is used to improve the uniformity of the surface temperature for the EB. Your assignment is to size the heaters, MH and TH, by specifying their applied heat fluxes, q''_{mh} and $q''_{th}(\mathrm{W/m^2})$, and their radial extents, Δr_{mh} and Δr_{th}. The constraints on radial positioning of the heaters are imposed by manufacturing considerations and are shown in the schematic.



Use the finite-element method of *FEHT* to perform a conduction analysis on an axisymmetric EB of 340-mm diameter. The upper and lateral surfaces are exposed to the ambient fluid at $T_\infty = 25°C$ with a convection coefficient of $10\,\mathrm{W/m^2 \cdot K}$. The lower surface of the EB is adiabatic, except for the ring sectors with the uniform applied heat fluxes, q''_{mh} and q''_{th}.

(a) For an upper surface of 140°C, perform an overall energy balance on the EB to obtain an initial estimate for the applied heater fluxes. Assume that $q''_{mh} = q''_{th}$ and that each heater extends fully over the radial limits indicated schematically. Use this estimate as a boundary condition in your *FEHT* model; determine the temperature distribution; and using the *View/Temperature Contours* command, examine the isotherms and the temperature distribution across the upper surface of the EB. Did you achieve the desired uniformity?

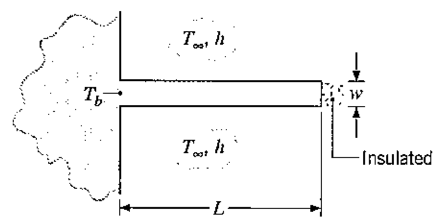
(b) Rerun your *FEHT* model with different values of the heater fluxes, until you obtain the best uniformity possible within the imposed constraints. *Hint*: If you want to know the specific nodal temperatures, see the *View/Tabular Output* page. You may obtain a plot of the surface temperature distribution by highlighting the appropriate nodal data, using the *Copy* command, and pasting the data into *Excel* for graphing.

(c) In what manner would a nonuniform distribution of the convection coefficient across the upper surface of the EB affect the temperature uniformity? For the downward flowing gas stream used in the microlithography process, a representative distribution of the convection coefficient on the upper surface of the EB is $h(r) = h_o[1 + a(r/r_o)^n]$, where $h_o = 5.4\,\mathrm{W/m^2 \cdot K}$ and $a = n = 1.5$. For this distribution and retention of a value of $h = 10\,\mathrm{W/m^2 \cdot K}$ at the lateral surface of the EB, can you adjust the trim heater flux to obtain improved uniformity of the surface temperature?

(d) What changes to the design would you propose for improving the surface temperature uniformity?

Special Applications

4.72 A straight fin of uniform cross section is fabricated from a material of thermal conductivity $50\,\mathrm{W/m \cdot K}$, thickness $w = 6\,\mathrm{mm}$, and length $L = 48\,\mathrm{mm}$, and is very long in the direction normal to the page. The convection heat transfer coefficient is $500\,\mathrm{W/m^2 \cdot K}$ with an ambient air temperature of $T_\infty = 30°C$. The base of the fin is maintained at $T_b = 100°C$, while the fin tip is well insulated.

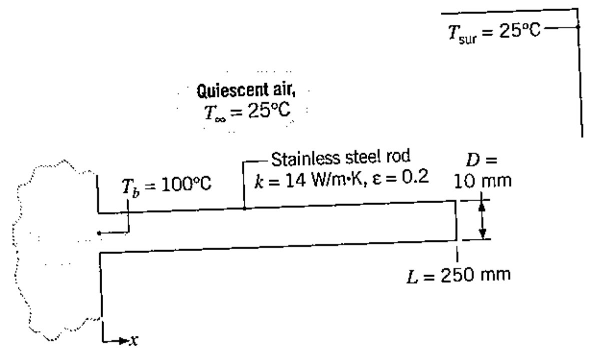


(a) Using a finite-difference method with a space increment of 4 mm, estimate the temperature distribution within the fin. Is the assumption of one-dimensional heat transfer reasonable for this fin?

(b) Estimate the fin heat transfer rate per unit length normal to the page. Compare your result with the one-dimensional, analytical solution, Equation 3.76.

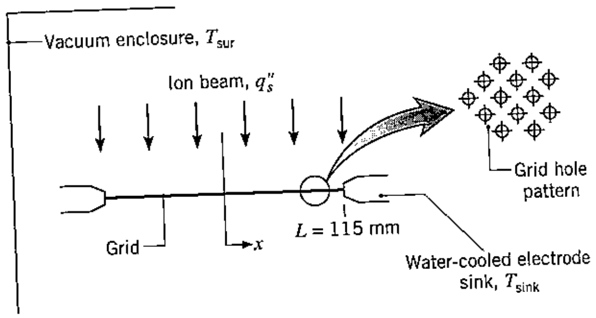
(c) Using the finite-difference mesh of part (a), compute and plot the fin temperature distribution for values of h = 10, 100, 500, and 1000 $W/m^2 \cdot K$. Determine and plot the fin heat transfer rate as a function of h.

4.73 A rod of 10-mm diameter and 250-mm length has one end maintained at 100°C. The surface of the rod experiences free convection with the ambient air at 25°C and a convection coefficient that depends on the difference between the temperature of the surface and the ambient air. Specifically, the coefficient is prescribed by a correlation of the form, $h_{fc} = 2.89[0.6 + 0.624(T - T_\infty)^{1/6}]^2$, where the units are h_{fc} ($W/m^2 \cdot K$) and T (K). The surface of the rod has an emissivity $\varepsilon = 0.2$ and experiences radiation exchange with the surroundings at $T_{sur} = 25°C$. The fin tip also experiences free convection and radiation exchange.



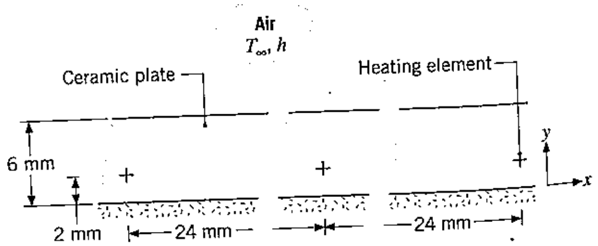
Assuming one-dimensional conduction and using a finite-difference method representing the fin by five nodes, estimate the temperature distribution for the fin. Determine also the fin heat rate and the relative contributions of free convection and radiation exchange. *Hint:* For each node requiring an energy balance, use the linearized form of the radiation rate equation, Equation 1.8, with the radiation coefficient h_r, Equation 1.9, evaluated for each node. Similarly, for the convection rate equation associated with each node, the free convection coefficient h_{fc} must be evaluated for each node.

4.74 A thin metallic foil of thickness 0.25 mm with a pattern of extremely small holes serves as an acceleration grid to control the electrical potential of an ion beam. Such a grid is used in a chemical vapor deposition (CVD) process for the fabrication of semiconductors. The top surface of the grid is exposed to a uniform heat flux caused by absorption of the ion beam, $q''_s = 600\ W/m^2$. The edges of the foil are thermally coupled to water-cooled sinks maintained at 300 K. The upper and lower surfaces of the foil experience radiation exchange with the vacuum enclosure walls maintained at 300 K. The effective thermal conductivity of the foil material is 40 W/m · K and its emissivity is 0.45.



Assuming one-dimensional conduction and using a finite-difference method representing the grid by ten nodes in the x direction, estimate the temperature distribution for the grid. *Hint:* For each node requiring an energy balance, use the linearized form of the radiation rate equation, Equation 1.8, with the radiation coefficient h_r, Equation 1.9, evaluated for each node.

4.75 Small diameter electrical heating elements dissipating 50 W/m (length normal to the sketch) are used to heat a ceramic plate of thermal conductivity 2 W/m · K. The upper surface of the plate is exposed to ambient air at 30°C with a convection coefficient of 100 $W/m^2 \cdot K$, while the lower surface is well insulated.



(a) Using the Gauss–Seidel method with a grid spacing of Δx = 6 mm and Δy = 2 mm, obtain the temperature distribution within the plate.

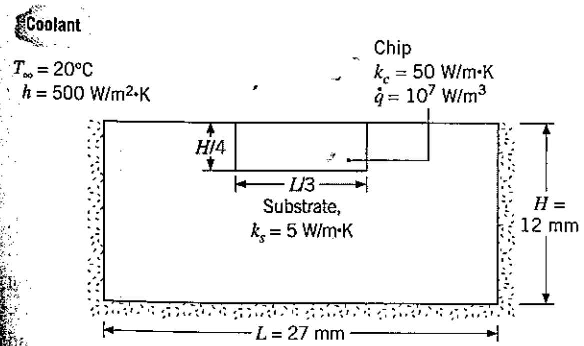
(b) Using the calculated nodal temperatures, sketch four isotherms to illustrate the temperature distribution in the plate.

(c) Calculate the heat loss by convection from the plate to the fluid. Compare this value with the element dissipation rate.

(d) What advantage, if any, is there in not making $\Delta x = \Delta y$ for this situation?

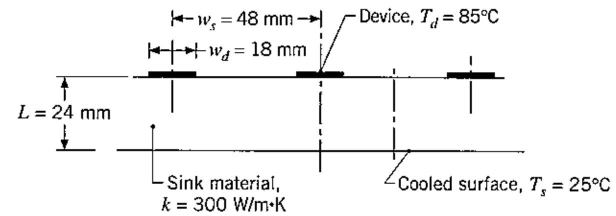
(e) With $\Delta x = \Delta y$ = 2 mm, calculate the temperature field within the plate and the rate of heat transfer from the plate. Under no circumstances may the temperature at any location in the plate exceed 400°C. Would this limit be exceeded if the air flow were terminated and heat transfer to the air was by natural convection with h = 10 $W/m^2 \cdot K$?

4.76 A simplified representation for cooling in very large-scale integration (VLSI) of microelectronics is shown in the sketch. A silicon chip is mounted in a dielectric substrate, and one surface of the system is convectively cooled, while the remaining surfaces are well insulated from the surroundings. The problem is rendered two-dimensional by assuming the system to be very long in the direction perpendicular to the paper. Under steady-state operation, electric power dissipation in the chip provides for uniform volumetric heating at a rate of $\dot{q}$. However, the heating rate is limited by restrictions on the maximum temperature that the chip is allowed to assume.



For the conditions shown on the sketch, will the maximum temperature in the chip exceed 85°C, the maximum allowable operating temperature set by industry standards? A grid spacing of 3 mm is suggested.

4.77 Electronic devices dissipating electrical power can be cooled by conduction to a heat sink. The lower surface of the sink is cooled, and the spacing of the devices w_s, the width of the device w_d, and the thickness L and thermal conductivity k of the heat sink material each affect the thermal resistance between the device and the cooled surface. The function of the heat sink is to *spread* the heat dissipated in the device throughout the sink material.



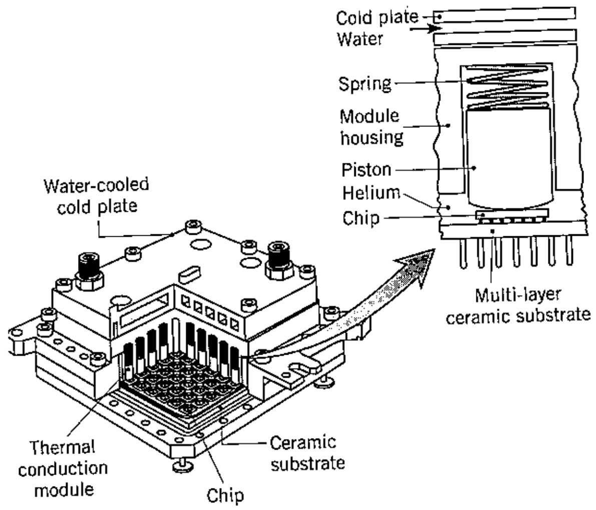
(a) Beginning with the shaded symmetrical element, use the flux plot method to estimate the thermal resistance per unit depth between the device and lower surface of the sink, $R'_{t,d-s}$ (m · K/W). How does this value compare with thermal resistances based on the assumption of one-dimensional conduction in rectangular domains of (i) width w_d and length L and (ii) width w_s and length L?

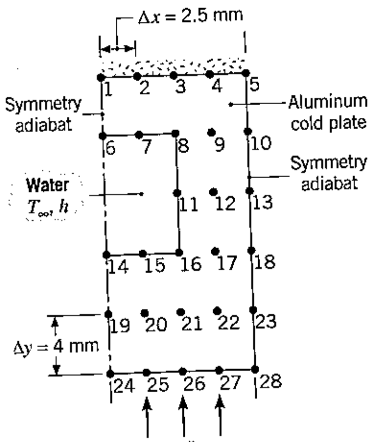
(b) Using a coarse (5 × 5) nodal network, calculate the thermal resistance $R'_{t,d-s}$ (m · K/W).

(c) Using nodal networks with grid spacings three and five times smaller than that in part (b), determine the effect of grid size on the precision of the thermal resistance calculation.

(d) Using the finer nodal network developed for part (c), determine the effect of device width on the thermal resistance. Specifically, keeping w_s and L fixed, find the thermal resistance for values of w_d/w_s = 0.175, 0.275, 0.375, and 0.475.

4.78 A major problem in packaging very large-scale integrated (VLSI) circuits concerns cooling of the circuit elements. The problem results from increasing levels of power dissipation within a chip, as well as from packing chips closer together in a module. A novel technique for cooling multichip modules has been developed by IBM. Termed the thermal conduction module (TCM), the chips are soldered to a multilayer ceramic substrate, and heat dissipated in each chip is conducted through a spring-loaded aluminum piston to a water-cooled *cold plate*.

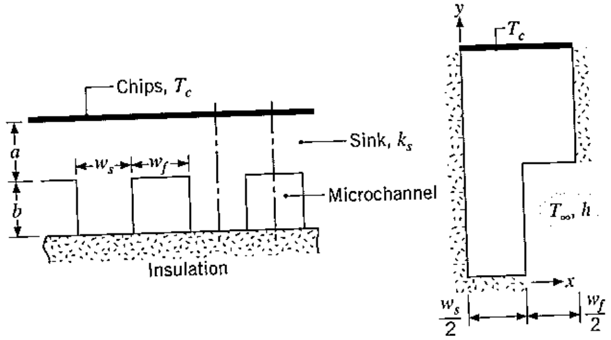




(a) Consider a cold plate fabricated from aluminum ($k = 190\,\mathrm{W/m \cdot K}$) with regularly spaced rectangular channels through which the water is passed. Under normal operation, power dissipation within the chips results in a uniform heat flux of $q''_o = 10^5\,\mathrm{W/m^2}$ at the base of the cold plate, while water flow provides a temperature of $T_\infty = 15°\mathrm{C}$ and a convection coefficient of $h = 5000\,\mathrm{W/m^2 \cdot K}$ within the channels. We are interested in obtaining the steady-state temperature distribution within the cold plate, and from symmetry considerations, we may confine our attention to the prescribed nodal network. Assuming the top surface of the cold plate to be well insulated, determine the nodal temperatures.

(b) Although there is interest in operating at higher power levels, system reliability considerations dictate that the maximum cold plate temperature must not exceed 40°C. Using the prescribed cold plate geometry and nodal network, assess the effect of changes in operating or design conditions intended to increase the operating heat flux q''_o. Estimate the upper limit for the heat flux.

4.79 A heat sink for cooling computer chips is fabricated from copper ($k_s = 400\,\mathrm{W/m \cdot K}$), with machined microchannels passing a cooling fluid for which $T_\infty = 25°\mathrm{C}$ and $h = 30{,}000\,\mathrm{W/m^2 \cdot K}$. The lower side of the sink experiences no heat removal, and a preliminary heat sink design calls for dimensions of $a = b = w_s = w_f = 200\,\mu\mathrm{m}$. A symmetrical element of the heat path from the chip to the fluid is shown in the inset.

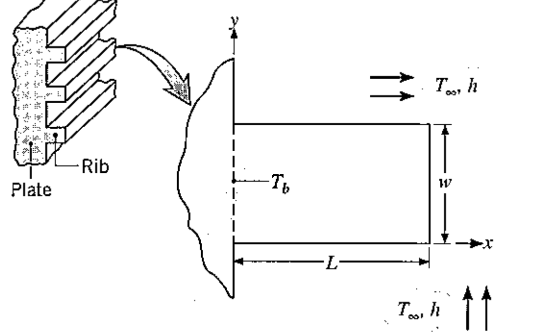


(a) Using the symmetrical element with a square nodal network of $\Delta x = \Delta y = 100\,\mu\mathrm{m}$, determine the corresponding temperature field and the heat rate q' to the coolant per unit channel length (W/m) for a maximum allowable chip temperature $T_{c,\max} = 75°\mathrm{C}$. Estimate the corresponding thermal resistance between the chip surface and the fluid, $R'_{t,c-f}$ (m · K/W). What is the maximum allowable heat dissipation for a chip that measures 10 mm × 10 mm on a side?

(b) The grid spacing used in the foregoing finite-difference solution is coarse, resulting in poor precision for the temperature distribution and heat removal rate. Investigate the effect of grid spacing by considering spatial increments of 50 and 25 μm.

(c) Consistent with the requirement that $a + b = 400\,\mu\mathrm{m}$, can the heat sink dimensions be altered in a manner that reduces the overall thermal resistance?

4.80 A plate ($k = 190\,\mathrm{W/m \cdot K}$) is stiffened by a series of longitudinal ribs having a rectangular cross section with length $L = 8$ mm and width $w = 4$ mm. The base of the plate is maintained at a uniform temperature $T_b = 45°\mathrm{C}$, while the rib surfaces are exposed to air at a temperature of $T_\infty = 25°\mathrm{C}$ and a convection coefficient of $h = 600\,\mathrm{W/m^2 \cdot K}$.

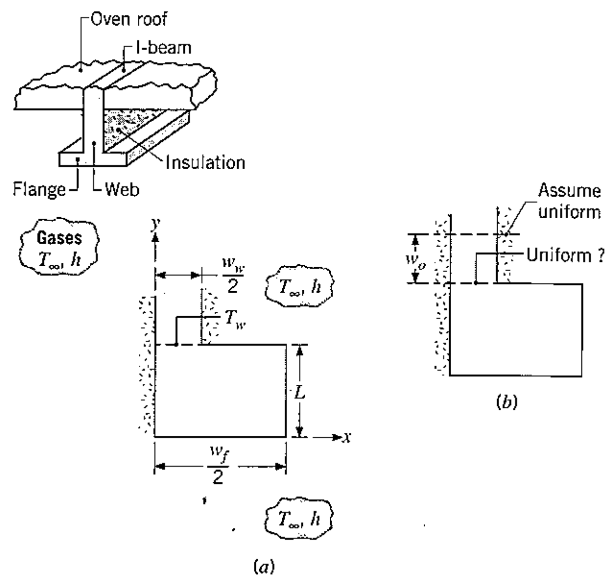


(a) Using a finite-difference method with $\Delta x = \Delta y = 2$ mm and a total of 5 × 3 nodal points and regions, estimate the temperature distribution and the heat rate from the base. Compare these results with those obtained by assuming that heat transfer in the rib is one-dimensional, thereby approximating the behavior of a fin.

(b) The grid spacing used in the foregoing finite-difference solution is coarse, resulting in poor precision for estimates of temperatures and the heat rate. Investigate the effect of grid refinement by reducing the nodal spacing to $\Delta x = \Delta y = 1$ mm (a 9 × 3 grid) considering symmetry of the center line.

(c) Investigate the nature of two-dimensional conduction in the rib and determine a criterion for which the one-dimensional approximation is reasonable. Do so by extending your finite-difference analysis to determine the heat rate from the base as a function of the length of the rib for the range $1.5 \le L/w \le 10$, keeping the length L constant. Compare your results with those determined by approximating the rib as a fin.

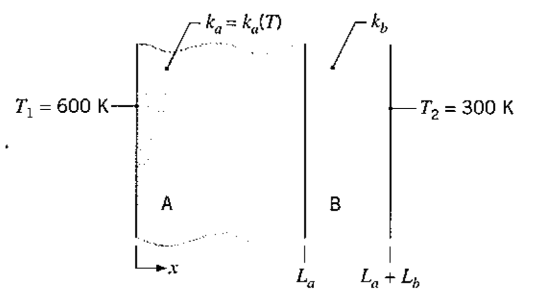
4.81 The bottom-half of an I-beam providing support for a furnace roof extends into the heating zone. The web is well insulated, while the flange surfaces experience convection with hot gases at $T_\infty = 400°\mathrm{C}$ and a convection coefficient of $h = 150\,\mathrm{W/m^2 \cdot K}$. Consider the symmetrical element of the flange region (inset a), assuming that the temperature distribution across the web is uniform at $T_w = 100°\mathrm{C}$. The beam thermal conductivity is $10\,\mathrm{W/m \cdot K}$, and its dimensions are $w_f = 80$ mm, $w_w = 30$ mm, and $L = 30$ mm.



(a) Calculate the heat transfer rate per unit length to the beam using a 5 × 4 nodal network.

(b) Is it reasonable to assume that the temperature distribution across the web–flange interface is uniform? Consider the L-shaped domain of inset (b) and use a fine grid to obtain the temperature distribution across the web–flange interface. Make the distance $w_o \ge w_w/2$.

4.82 Consider one-dimensional conduction in a plane composite wall. The exposed surfaces of materials A and B are maintained at $T_1 = 600$ K and $T_2 = 300$ K respectively. Material A, of thickness $L_a = 20$ mm, has a temperature-dependent thermal conductivity of $k_a = k_o[1 + \alpha(T - T_o)]$, where $k_o = 4.4\,\mathrm{W/m \cdot K}$, $\alpha = 0.008\,\mathrm{K^{-1}}$, $T_o = 300$ K and T is in kelvins. Material B is of thickness $L_b = 5$ mm and has a thermal conductivity of $k_b = 1\,\mathrm{W/m \cdot K}$.

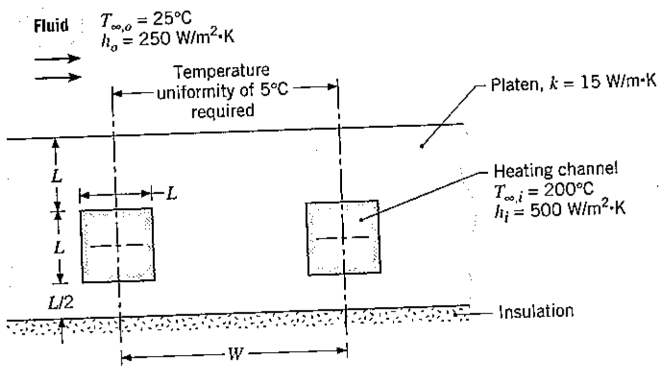


(a) Calculate the heat flux through the composite wall by assuming material A to have a uniform thermal conductivity evaluated at the average temperature of the section.

(b) Using a space increment of 1 mm, obtain the finite-difference equations for the internal nodes and calculate the heat flux considering the temperature-dependent thermal conductivity for Material A. If the *IHT* software is employed, call-up functions from *Tools/Finite-Difference Equations* may be used to obtain the nodal equations. Compare your result with that obtained in Part (a).

(c) As an alternative to the finite-difference method of Part (b), use the finite-element method of *FEHT* to calculate the heat flux, and compare the result with that from Part (a). *Hint*: In the *Specify/Material Properties* box, properties may be entered as a function of temperature (T), the space coordinates (x, y), or time (t). See the *Help* section for more details.

4.83 A platen of thermal conductivity $k = 15\ \text{W/m}\cdot\text{K}$ is heated by flow of a hot fluid through channels of width $L = 20\ \text{mm}$, with $T_{\infty,i} = 200°\text{C}$ and $h_i = 500\ \text{W/m}^2\cdot\text{K}$. The upper surface of the platen is used to heat a process fluid at $T_{\infty,o} = 25°\text{C}$ with a convection coefficient of $h_o = 250\ \text{W/m}^2\cdot\text{K}$. The lower surface of the platen is insulated. To heat the process fluid uniformly, the temperature of the platen's upper surface must be uniform to within 5°C. Use a finite-difference method, such as that of *IHT*, or the finite-element method of *FEHT* to obtain the following results.



(a) Determine the maximum allowable spacing, W, between the channel centerlines that will satisfy the specified temperature uniformity requirement.

(b) What is the corresponding heat rate per unit length from a flow channel?

4.84 Consider the cooling arrangement for the very large-scale integration (VLSI) chip of Problem 4.77. Use the finite-element method of *FEHT* to obtain the following results.

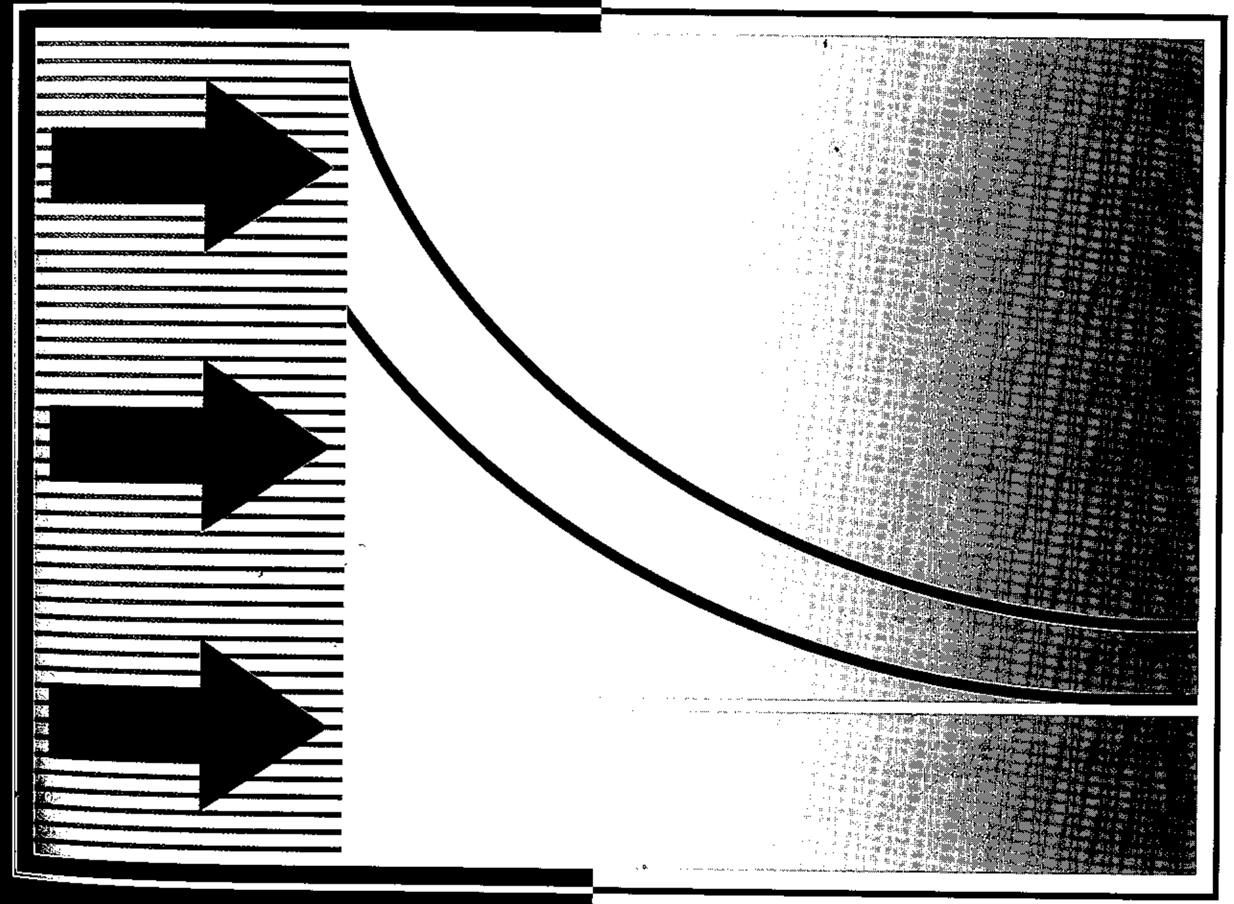
(a) Determine the temperature distribution in the chip-substrate system. Will the maximum temperature exceed 85°C?

(b) Using the *FEHT* model developed for Part (a), determine the volumetric heating rate that yields a maximum temperature of 85°C?

(c) What effect would reducing the substrate thickness have on the maximum operating temperature? For a volumetric generation rate of $\dot{q} = 10^7\ \text{W/m}^3$, reduce the thickness of the substrate from 12 mm to 6 mm, keeping all other dimensions unchanged. What is the maximum system temperature for these conditions? What fraction of the chip power generation is removed by convection directly from the chip surface?

CHAPTER 5

Transient Conduction



Note that the prescribed convection coefficient renders convection negligible, as required by the problem statement. Using $\Delta x = 75$ mm and $\Delta t = 12$ s, the solution yields $T_0 = T(0,120\text{ s}) = 116.0°C$ and $T_2 = T(150\text{ mm}, 120\text{ s}) = 44.2°C$. Contrast these results with those summarized in Comment 1.

6. This example is provided in *FEHT* as a solved model accessed through the *Toolbar* menu, *Examples*. The input screen summarizes key pre- and post-processing steps, as well as results for nodal spacings of 1 and 0.125 mm. As an exercise, press *Run* to solve for the nodal temperatures, and in the *View* menu, select *Temperature Contours* to represent the temperature field in the form of isotherms.

5.10 Summary

Transient conduction occurs in numerous engineering applications and may be treated using different methods. There is certainly much to be said for simplicity, in which case, when confronted with a transient problem, the first thing you should do is calculate the Biot number. If this number is much less than unity, you may use the lumped capacitance method to obtain accurate results with minimal computational requirements. However, if the Biot number is not much less than unity, spatial effects must be considered, and some other method must be used. Analytical results are available in convenient graphical and equation form for the plane wall, the infinite cylinder, the sphere, and the semi-infinite solid. You should know when and how to use these results. If geometrical complexities and/or the form of the boundary conditions preclude their use, recourse must be made to an approximate numerical technique, such as the finite-difference method.

You may test your understanding of key concepts by addressing the following questions.

- Under what conditions may the *lumped capacitance method* be used to predict the transient response of a solid to a change in its thermal environment?
- What is the physical interpretation of the *Biot number*?
- Is the lumped capacitance method of analysis likely to be more applicable for a hot solid being cooled by forced convection in air or in water? By forced convection in air or natural convection in air?
- Is the lumped capacitance method of analysis likely to be more applicable for cooling of a hot solid made of copper or aluminum? For silicon nitride or glass?
- What parameters determine the *time constant* associated with the transient thermal response of a lumped capacitance solid? Is this response accelerated or decelerated by an increase in the convection coefficient? By an increase in the density or specific heat of the solid?

- For one-dimensional, transient conduction in a plane wall, a long cylinder or a sphere with surface convection, what dimensionless parameters may be used to simplify the representation of thermal conditions? How are these parameters defined?
- What is the physical interpretation of the *Fourier number*?
- What requirement must be satisfied for use of a *one-term approximation* to determine the transient thermal response of a plane wall, a long cylinder or a sphere experiencing one-dimensional conduction due to a change in surface conditions? At what stage of a transient process is the requirement not satisfied?
- What does transient heating or cooling of a plane wall with equivalent convection conditions at opposite surfaces have in common with a plane wall heated or cooled by convection at one surface and well insulated at the other surface?
- How may a one-term approximation be used to determine the transient thermal response of a plane wall, long cylinder, or sphere subjected to a sudden change in surface temperature?
- For one-dimensional, transient conduction, what is implied by the idealization of a *semi-infinite* solid? Under what conditions may the idealization be applied to a plane wall?
- What differentiates an *explicit*, finite-difference solution to a transient conduction problem from an *implicit* solution?
- What is meant by characterization of the implicit finite-difference method *as unconditionally stable*? What constraint is placed on the explicit method to ensure a stable solution?

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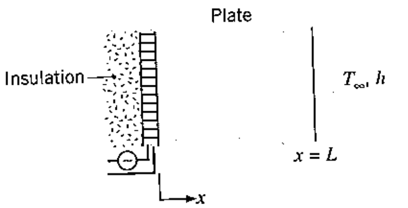
Problems

Qualitative Considerations

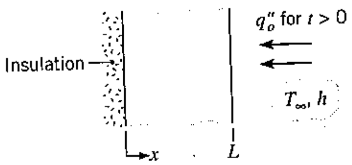
5.1 Consider a thin electrical heater attached to a plate and backed by insulation. Initially, the heater and plate are at the temperature of the ambient air, T_∞. Suddenly, the power to the heater is activated, yielding a constant heat flux q''_o (W/m²) at the inner surface of the plate.



(a) Sketch and label, on T–x coordinates, the temperature distributions: initial, steady-state, and at two intermediate times.

(b) Sketch the heat flux at the outer surface $q''_x(L, t)$ as a function of time.

5.2 The inner surface of a plane wall is insulated while the outer surface is exposed to an airstream at T_∞. The wall is at a uniform temperature corresponding to that of the airstream. Suddenly, a radiation heat source is switched on applying a uniform flux q''_o to the outer surface.



(a) Sketch and label, on T–x coordinates, the temperature distributions: initial, steady-state, and at two intermediate times.

(b) Sketch the heat flux at the outer surface $q''_x(L, t)$ as a function of time.

5.3 A microwave oven operates on the principle that application of a high-frequency field causes electrically polarized molecules in food to oscillate. The net effect is a nearly *uniform generation* of thermal energy within the food. Consider the process of cooking a slab of beef of thickness $2L$ in a microwave oven and compare it with cooking in a conventional oven, where *each side* of the slab is *heated by radiation*. In each case the meat is to be heated from 0°C to a *minimum* temperature of 90°C. Base your comparison on a sketch of the temperature distribution at selected times for each of the cooking processes. In particular, consider the time t_0 at which heating is initiated, a time t_1 during the heating process, the time t_2 corresponding to the conclusion of heating, and a time t_3 well into the subsequent cooling process.

5.4 A plate of thickness $2L$, surface area A_s, mass M, and specific heat c_p, initially at a uniform temperature T_i, is suddenly heated on both surfaces by a convection process (T_∞, h) for a period of time t_o, following which the plate is insulated. Assume that the midplane temperature does not reach T_∞ within this period of time.

(a) Assuming $Bi \gg 1$ for the heating process, sketch and label, on T–x coordinates, the following temperature distributions: initial, steady-state ($t \to \infty$), $T(x, t_o)$, and at two intermediate times between $t = t_o$ and $t \to \infty$.

(b) Sketch and label, on T–t coordinates, the midplane and exposed surface temperature distributions.

(c) Repeat parts (a) and (b) assuming $Bi \ll 1$ for the plate.

(d) Derive an expression for the steady-state temperature $T(x, \infty) = T_f$, leaving your result in terms of plate parameters (M, c_p), thermal conditions (T_i, T_∞, h), the surface temperature $T(L, t)$, and the heating time t_o.

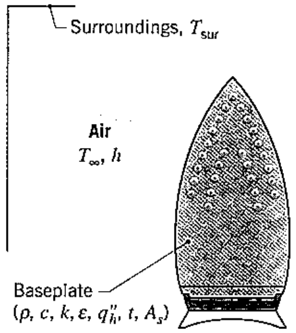
Lumped Capacitance Method

5.5 Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_\infty = 325$ K and $h = 20$ W/m$^2 \cdot$K. Assuming the properties of the steel to be $k = 40$ W/m $\cdot$ K, $\rho = 7800$ kg/m^3, and $c = 600$ J/kg $\cdot$ K, estimate the time required for the cooling process.

5.6 The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature–time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an airstream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C 69 s after the sphere is inserted in the airstream. Assume, and then justify, that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient.

5.7 A solid steel sphere (AISI 1010), 300 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/m $\cdot$ K. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which $T_\infty = 100$°C and $h = 3300$ W/m$^2 \cdot$ K. Estimate the time required for the coated sphere temperature to reach 140°C. *Hint:* Neglect the effect of energy storage in the dielectric material, since its thermal capacitance (ρcV) is small compared to that of the steel sphere.

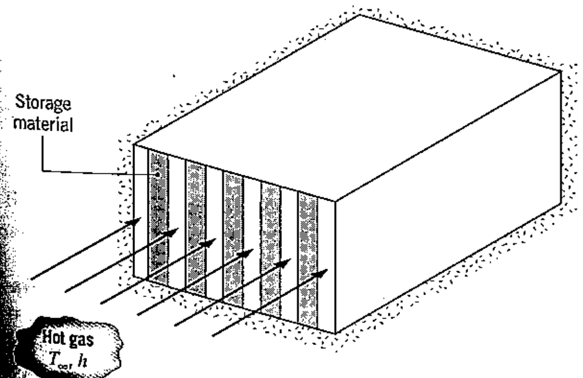
5.8 The base plate of an iron has a thickness of $L = 7$ mm and is made from an aluminum alloy ($\rho = 2800$ kg/m^3, $c = 900$ J/kg $\cdot$ K, $k = 180$ W/m $\cdot$ K, $\varepsilon = 0.80$). An electric resistance heater is attached to the inner surface of the plate, while the outer surface is exposed to ambient air and large surroundings at $T_\infty = T_{sur} = 25$°C. The areas of both the inner and outer surfaces are $A_s = 0.040$ m^2.



If an approximately uniform heat flux of $q''_h = 1.25 \times 10^4$ W/m^2 is applied to the inner surface of the base plate and the convection coefficient at the outer surface is $h = 10$ W/m$^2 \cdot$ K, estimate the time required for the plate to reach a temperature of 135°C.

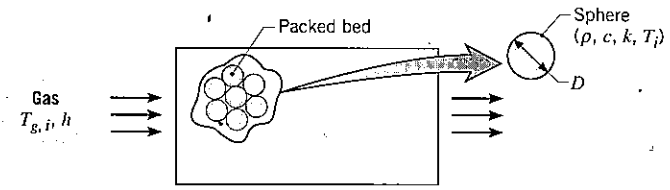
5.9 Carbon steel (AISI 1010) shafts of 0.1-m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of 100 W/m$^2 \cdot$ K. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?

5.10 A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage.



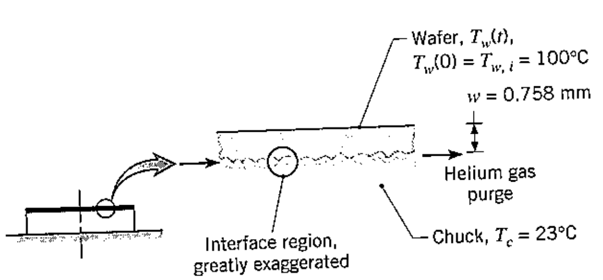
Each layer of the storage material is an aluminum slab of width $W = 0.05$ m, which is at an initial temperature of 25°C. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of $T_\infty = 600$°C and $h = 100$ W/m$^2 \cdot$ K throughout the channel. How long will it take to achieve 75% of the maximum possible energy storage? What is the temperature of the aluminum at this time?

5.11 Thermal energy storage systems commonly involve a *packed bed* of solid spheres, through which a hot gas flows if the system is being charged or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas.



Consider a packed bed of 75-mm-diameter aluminum spheres ($\rho = 2700$ kg/m^3, $c = 950$ J/kg $\cdot$ K, $k = 240$ W/m $\cdot$ K) and a charging process for which gas enters the storage unit at a temperature of $T_{g,i} = 300$°C. If the initial temperature of the spheres is $T_i = 25$°C and the convection coefficient is $h = 75$ W/m$^2 \cdot$ K, how long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere? Is there any advantage to using copper instead of aluminum?

5.12 A tool used for fabricating semiconductor devices consists of a chuck (thick metallic, cylindrical disk) onto which a very thin silicon wafer ($\rho = 2700$ kg/m^3, $c = 875$ J/kg $\cdot$ K, $k = 177$ W/m $\cdot$ K) is placed by a robotic arm. Once in position, an electric field in the chuck is energized, creating an electrostatic force that holds the wafer firmly to the chuck. To ensure a reproducible thermal contact resistance between the chuck and the wafer from cycle-to-cycle, pressurized helium gas is introduced at the center of the chuck and flows (very slowly) radially outward between the asperites of the interface region.



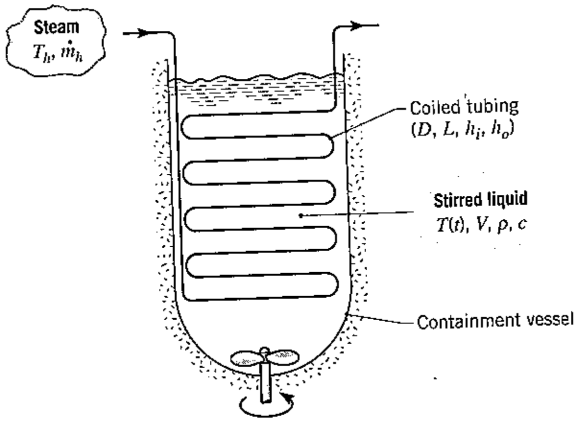
An experiment has been performed under conditions for which the wafer, initially at a uniform temperature $T_{w,i} = 100°C$, is suddenly placed on the chuck, which is at a uniform and constant temperature $T_c = 23°C$. With the wafer in place, the electrostatic force and the helium gas flow are applied. After 15 seconds, the temperature of the wafer is determined to be 33°C. What is the thermal contact resistance $R''_{t,c}$ (m² · K/W) between the wafer and chuck? Will the value of $R''_{t,c}$ increase, decrease, or remain the same if air, instead of helium, is used as the purge gas?

5.13 A spherical vessel used as a reactor for producing pharmaceuticals has a 5-mm-thick stainless steel wall ($k = 17$ W/m · K) and an inner diameter of $D_i = 1.0$ m. During production, the vessel is filled with reactants for which $\rho = 1100$ kg/m³ and $c = 2400$ J/kg · K, while exothermic reactions release energy at a volumetric rate of $\dot{q} = 10^4$ W/m³. As first approximations, the reactants may be assumed to be well stirred and the thermal capacitance of the vessel may be neglected.

(a) The exterior surface of the vessel is exposed to ambient air ($T_\infty = 25°C$) for which a convection coefficient of $h = 6$ W/m² · K may be assumed. If the initial temperature of the reactants is 25°C, what is the temperature of the reactants after five hours of process time? What is the corresponding temperature at the outer surface of the vessel?

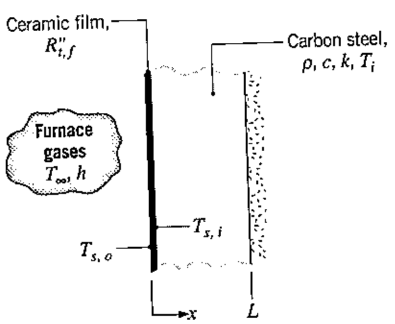
(b) Explore the effect of varying the convection coefficient on transient thermal conditions within the reactor.

5.14 Batch processes are often used in chemical and pharmaceutical operations to achieve a desired chemical composition for the final product and typically involve a transient heating operation to take the product from room temperature to the desired process temperature. Consider a situation for which a chemical of density $\rho = 1200$ kg/m³ and specific heat $c = 2200$ J/kg · K occupies a volume of $V = 2.25$ m³ in an insulated vessel. The chemical is to be heated from room temperature, $T_i = 300$ K, to a process temperature of $T = 450$ K by passing saturated steam at $T_h = 500$ K through a coiled, thin-walled, 20-mm-diameter tube in the vessel. Steam condensation within the tube maintains an interior convection coefficient of $h_i = 10{,}000$ W/m² · K, while the highly agitated liquid in the stirred vessel maintains an outside convection coefficient of $h_o = 2000$ W/m² · K.



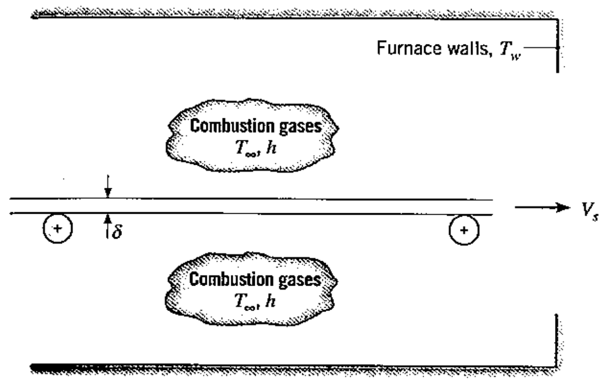
If the chemical is to be heated from 300 to 450 K in 60 minutes, what is the required length L of the submerged tubing?

5.15 A plane wall of a furnace is fabricated from plain carbon steel ($k = 60$ W/m · K, $\rho = 7850$ kg/m³, $c = 430$ J/kg · K) and is of thickness $L = 10$ mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f} = 0.01$ m² · K/W. The opposite surface is well insulated from the surroundings.



At furnace start-up the wall is at an initial temperature of $T_i = 300$ K, and combustion gases at $T_\infty = 1300$ K enter the furnace, providing a convection coefficient of $h = 25$ W/m² · K at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of $T_{s,i} = 1200$ K? What is the temperature $T_{s,o}$ of the exposed surface of the ceramic film at this time?

5.16 A steel strip of thickness $\delta = 12$ mm is annealed by passing it through a large furnace whose walls are maintained at a temperature T_w corresponding to that of combustion gases flowing through the furnace ($T_w = T_\infty$). The strip, whose density, specific heat, thermal conductivity, and emissivity are $\rho = 7900$ kg/m³, $c_p = 640$ J/kg · K, $k = 30$ W/m · K, and $\varepsilon = 0.7$, respectively, is to be heated from 300°C to 600°C.



(a) For a uniform convection coefficient of $h = 100$ W/m² · K and $T_w = T_\infty = 700°C$, determine the time required to heat the strip. If the strip is moving at 0.5 m/s, how long must the furnace be?

(b) The annealing process may be accelerated (the strip speed increased) by increasing the environmental temperatures. For the furnace length obtained in part (a), determine the strip speed for $T_w = T_\infty = 850°C$ and $T_w = T_\infty = 1000°C$. For each set of environmental temperatures (700, 850, and 1000°C), plot the strip temperature as a function of time over the range $25°C \le T \le 600°C$. Over this range, also plot the radiation heat transfer coefficient, h_r, as a function of time.

5.17 A long wire of diameter $D = 1$ mm is submerged in an oil bath of temperature $T_\infty = 25°C$. The wire has an electrical resistance per unit length of $R'_e = 0.01$ Ω/m. If a current of $I = 100$A flows through the wire and the convection coefficient is $h = 500$ W/m² · K, what is the steady-state temperature of the wire? From the time the current is applied, how long does it take for the wire to reach a temperature that is within 1°C of the steady-state value? The properties of the wire are $\rho = 8000$ kg/m³, $c = 500$ J/kg · K, and $k = 20$ W/m · K.

5.18 Consider the system of Problem 5.1 where the temperature of the plate is spacewise isothermal during the transient process.

(a) Obtain an expression for the temperature of the plate as a function of time $T(t)$ in terms of q''_o, T_∞, h, L, and the plate properties ρ and c.

(b) Determine the thermal time constant and the steady-state temperature for a 12-mm-thick plate of pure copper when $T_\infty = 27°C$, $h = 50$ W/m² · K, and $q''_o = 5000$ W/m². Estimate the time required to reach steady-state conditions.

(c) For the conditions of part (b), as well as for $h = 100$ and 200 W/m² · K, compute and plot the corresponding temperature histories of the plate for $0 \le t \le 2500$ s.

5.19 An electronic device, such as a power transistor mounted on a finned heat sink, can be modeled as a spatially isothermal object with internal heat generation and an external convection resistance.

(a) Consider such a system of mass M, specific heat c, and surface area A_s, which is initially in equilibrium with the environment at T_∞. Suddenly, the electronic device is energized such that a constant heat generation $\dot{E}_g$ (W) occurs. Show that the temperature response of the device is

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{RC}\right)$$

where $\theta \equiv T - T(\infty)$ and $T(\infty)$ is the steady-state temperature corresponding to $t \to \infty$; $\theta_i = T_i - T(\infty)$; T_i = initial temperature of device; R = thermal resistance $1/\bar{h}A_s$; and C = thermal capacitance Mc.

(b) An electronic device, which generates 60 W of heat, is mounted on an aluminum heat sink weighing 0.31 kg and reaches a temperature of 100°C in ambient air at 20°C under steady-state conditions. If the device is initially at 20°C, what temperature will it reach 5 min after the power is switched on?

5.20 Before being injected into a furnace, pulverized coal is preheated by passing it through a cylindrical tube whose surface is maintained at $T_{sur} = 1000°C$. The coal pellets are suspended in an airflow and are known to move with a speed of 3 m/s. If the pellets may be approximated as spheres of 1-mm diameter and it may be assumed that they are heated by radiation transfer from the tube surface, how long must the tube be to heat coal entering at 25°C to a temperature of 600°C? Is the use of the lumped capacitance method justified?

21 A metal sphere of diameter D, which is at a uniform temperature T_i, is suddenly removed from a furnace and suspended from a fine wire in a large room with air at a uniform temperature T_∞ and the surrounding walls at a temperature T_{sur}.

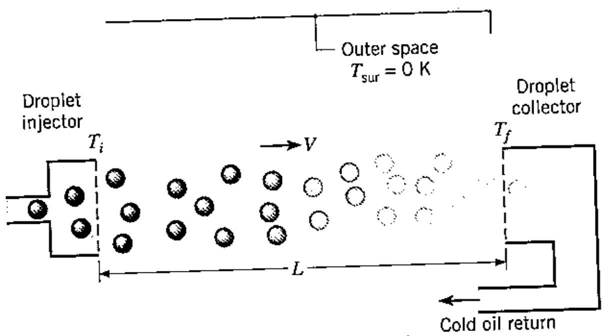
(a) Neglecting heat transfer by radiation, obtain an expression for the time required to cool the sphere to some temperature T.

(b) Neglecting heat transfer by convection, obtain an expression for the time required to cool the sphere to the temperature T.

(c) How would you go about determining the time required for the sphere to cool to the temperature T if both convection and radiation are of the same order of magnitude?

(d) Consider an anodized aluminum sphere (ε = 0.75) 50 mm in diameter, which is at an initial temperature of T_i = 800 K. Both the air and surroundings are at 300 K, and the convection coefficient is 10 W/m$^2\cdot$K. For the conditions of parts (a), (b), and (c), determine the time required for the sphere to cool to 400 K. Plot the corresponding temperature histories. Repeat the calculations for a polished aluminum sphere (ε = 0.1).

5.22 As permanent space stations increase in size, there is an attendant increase in the amount of electrical power they dissipate. To keep station compartment temperatures from exceeding prescribed limits, it is necessary to transfer the dissipated heat to space. A novel heat rejection scheme that has been proposed for this purpose is termed a Liquid Droplet Radiator (LDR). The heat is first transferred to a high vacuum oil, which is then injected into outer space as a stream of small droplets. The stream is allowed to traverse a distance L, over which it cools by radiating energy to outer space at absolute zero temperature. The droplets are then collected and routed back to the space station.



Consider conditions for which droplets of emissivity ε = 0.95 and diameter D = 0.5 mm are injected at a temperature of T_i = 500 K and a velocity of V = 0.1 m/s. Properties of the oil are ρ = 885 kg/m^3, c = 1900 J/kg$\cdot$K, and k = 0.145 W/m$\cdot$K. Assuming each drop to radiate to deep space at T_{sur} = 0 K, determine the distance L required for the droplets to impact the collector at a final temperature of T_f = 300 K. What is the amount of thermal energy rejected by each droplet?

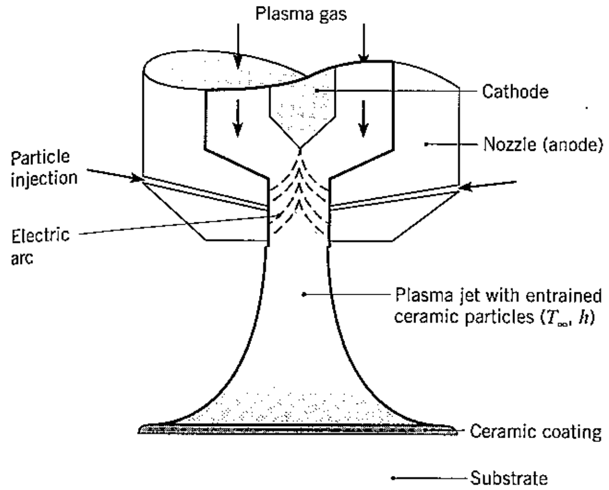
5.23 In a material processing experiment conducted aboard the space shuttle, a coated niobium sphere of 10-mm diameter is removed from a furnace at 900°C and cooled to a temperature of 300°C. Although properties of the niobium vary over this temperature range, constant values may be assumed to a reasonable approximation, with ρ = 8600 kg/m^3, c = 290 J/kg$\cdot$K, and k = 63 W/m$\cdot$K.

(a) If cooling is implemented in a large evacuated chamber whose walls are at 25°C, determine the time required to reach the final temperature if the coating is polished and has an emissivity of ε = 0.1. How long would it take if the coating is oxidized and ε = 0.6?

(b) To reduce the time required for cooling, consideration is given to immersion of the sphere in an inert gas stream for which T_∞ = 25°C and h = 200 W/m$^2\cdot$K. Neglecting radiation, what is the time required for cooling?

(c) Considering the effect of both radiation and convection, what is the time required for cooling if h = 200 W/m$^2\cdot$K and ε = 0.6? Explore the effect on the cooling time of independently varying h and ε.

5.24 Plasma spray-coating processes are often used to provide surface protection for materials exposed to hostile environments, which induce degradation through factors such as wear, corrosion, or outright thermal failure. *Ceramic* coatings are commonly used for this purpose. By injecting ceramic powder through the nozzle (anode) of a plasma torch, the particles are entrained by the plasma jet, within which they are then accelerated and heated.



During their *time-in-flight*, the ceramic particles must be heated to their melting point and experience complete conversion to the liquid state. The coating is formed as the molten droplets impinge (*splat*) on the substrate material and experience rapid solidification. Consider conditions for which spherical alumina (Al_2O_3) particles of diameter D_p = 50 μm, density ρ_p = 3970 kg/m^3, thermal conductivity k_p = 10.5 W/m$\cdot$K, and specific heat c_p = 1560 J/kg$\cdot$K are injected into an arc plasma, which is at T_∞ = 10,000 K and provides a coefficient of h = 30,000 W/m$^2\cdot$K for convective heating of the particles. The melting point and latent heat of fusion of alumina are T_{mp} = 2318 K and h_{sf} = 3577 kJ/kg, respectively.

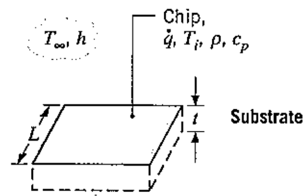
(a) Neglecting radiation, obtain an expression for the time-in-flight, t_{i-f}, required to heat a particle from its initial temperature T_i to its melting point T_{mp}, and, once at the melting point, for the particle to experience complete melting. Evaluate t_{i-f} for T_i = 300 K and the prescribed heating conditions.

(b) Assuming alumina to have an emissivity of ε_p = 0.4 and the particles to exchange radiation with large surroundings at T_{sur} = 300 K, assess the validity of neglecting radiation.

5.25 Thin film coatings characterized by high resistance to abrasion and fracture may be formed by using microscale composite particles in a plasma spraying process. A spherical particle typically consists of a *ceramic core*, such as tungsten carbide (WC), and a *metallic shell*, such as cobalt (Co). The ceramic provides the thin film coating with its desired hardness at elevated temperatures, while the metal serves to coalesce the particles on the coated surface and to inhibit crack formation. In the plasma spraying process, the particles are injected into a plasma gas jet that heats them to a temperature above the melting point of the metallic casing and melts the casing before the particles impact the surface.

Consider spherical particles comprised of a WC core of diameter D_i = 16 μm, which is encased in a Co shell of outer diameter D_o = 20 μm. If the particles flow in a plasma gas at T_∞ = 10,000 K and the coefficient associated with convection from the gas to the particles is h = 20,000 W/m$^2\cdot$K, how long does it take to heat the particles from an initial temperature of T_i = 300 K to the melting point of cobalt, T_{mp} = 1770 K? The density and specific heat of WC (the core of the particle) are ρ_c = 16,000 kg/m^3 and c_c = 300 J/kg$\cdot$K, while the corresponding values for Co (the outer shell) are ρ_s = 8900 kg/m^3 and c_s = 750 J/kg$\cdot$K. Once having reached the melting point, how much additional time is required to completely melt the cobalt if its latent heat of fusion is h_{sf} = 2.59 × 10^5 J/kg? You may use the lumped capacitance method of analysis and neglect radiation exchange between the particle and its surroundings.

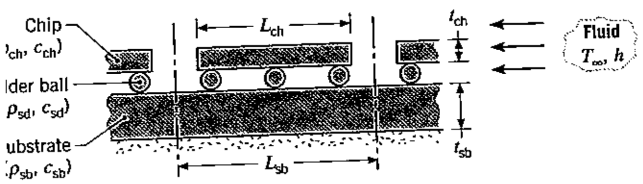
5.26 A chip that is of length L = 5 mm on a side and thickness t = 1 mm is encased in a ceramic substrate, and its exposed surface is convectively cooled by a dielectric liquid for which h = 150 W/m$^2\cdot$K and T_∞ = 20°C.



In the off-mode the chip is in thermal equilibrium with the coolant (T_i = T_∞). When the chip is energized, however, its temperature increases until a new steady-state is established. For purposes of analysis, the energized chip is characterized by uniform volumetric heating with $\dot q$ = 9 × 10^6 W/m^3. Assuming an infinite contact resistance between the chip and substrate and negligible conduction resistance within the chip, determine the steady-state chip temperature T_f. Following activation of the chip, how long does it take to come within 1°C of this temperature? The chip density and specific heat are $\rho = 2000\ \text{kg/m}^3$ and $c = 700\ \text{J/kg}\cdot\text{K}$, respectively.

5.27 Consider the conditions of Problem 5.26. In addition to treating heat transfer by convection directly from the chip to the coolant, a more realistic analysis would account for indirect transfer from the chip to the substrate and then from the substrate to the coolant. The total thermal resistance associated with this indirect route includes contributions due to the chip–substrate interface (a contact resistance), multidimensional conduction in the substrate, and convection from the surface of the substrate to the coolant. If this total thermal resistance is $R_t = 200$ K/W, what is the steady-state chip temperature T_f? Following activation of the chip, how long does it take to come within 1°C of this temperature?

5.28 Thermal stress testing is a common procedure used to assess the reliability of an electronic package. Typically, thermal stresses are induced in soldered or wired connections to reveal mechanisms that could cause failure and must therefore be corrected before the product is released. As an example of the procedure, consider an array of silicon chips ($\rho_{ch} = 2300\ \text{kg/m}^3$, $c_{ch} = 710\ \text{J/kg}\cdot\text{K}$) joined to an alumina substrate ($\rho_{sb} = 4000\ \text{kg/m}^3$, $c_{sb} = 770\ \text{J/kg}\cdot\text{K}$) by solder balls ($\rho_{sd} = 11{,}000\ \text{kg/m}^3$, $c_{sd} = 130\ \text{J/kg}\cdot\text{K}$). Each chip of width L_{ch} and thickness t_{ch} is joined to a unit substrate section of width L_{sb} and thickness t_{sb} by solder balls of diameter D.

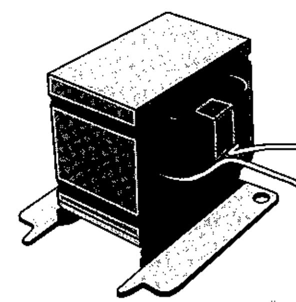


A thermal stress test begins by subjecting the multichip module, which is initially at room temperature, to a hot fluid stream and subsequently cooling the module by exposing it to a cold fluid stream. The process is repeated for a prescribed number of cycles to assess the integrity of the soldered connections.

(a) As a first approximation, assume that there is negligible heat transfer between the components (chip/solder/substrate) of the module and that the thermal response of each component may be determined from a lumped capacitance analysis involving the same convection coefficient h. Assuming no reduction in surface area due to contact between a solder ball and the chip or substrate, obtain expressions for the thermal time constant of each component. Heat transfer is to all surfaces of a chip, but to only the top surface of the substrate. Evaluate the three time constants for $L_{ch} = 15$ mm, $t_{ch} = 2$ mm, $L_{sb} = 25$ mm, $t_{sb} = 10$ mm, $D = 2$ mm, and a value of $h = 50\ \text{W/m}^2\cdot\text{K}$, which is characteristic of an air stream. Compute and plot the temperature histories of the three components for the heating portion of a cycle, with $T_i = 20°\text{C}$ and $T_\infty = 80°\text{C}$. At what time does each component experience 99% of its maximum possible temperature rise, that is, $(T - T_i)/(T_\infty - T_i) = 0.99$? If the maximum stress on a solder ball corresponds to the maximum difference between its temperature and that of the chip or substrate, when will this maximum occur?

(b) To reduce the time required to complete a stress test, a dielectric liquid could be used in lieu of air to provide a larger convection coefficient of $h = 200\ \text{W/m}^2\cdot\text{K}$. What is the corresponding savings in time for each component to achieve 99% of its maximum possible temperature rise?

5.29 The objective of this problem is to develop thermal models for estimating the steady-state temperature and the transient temperature history of the electrical transformer shown below.



The external transformer geometry is approximately cubical, with a length of 32 mm to a side. The combined mass of the iron and copper in the transformer is 0.28 kg, and its weighted-average specific heat is 400 J/kg · K. The transformer dissipates 4.0 W and is operating in ambient air at $T_\infty = 20°\text{C}$, with a convection coefficient of $10\ \text{W/m}^2\cdot\text{K}$. List and justify the assumptions made in your analysis, and discuss the limitations of the models.

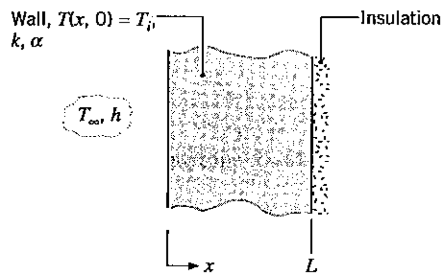
(a) Beginning with a properly defined control volume, develop a model for estimating the steady-state temperature of the transformer, $T(\infty)$. Evaluate $T(\infty)$ for the prescribed operating conditions.

(b) Develop a model for estimating the thermal response (temperature history) of the transformer if it is initially at a temperature of $T_i = T_\infty$ and power is suddenly applied. Determine the time required for the transformer to come within 5°C of its steady-state operating temperature.

One-Dimensional Conduction: The Plane Wall

5.30 Consider the series solution, Equation 5.39, for the plane wall with convection. Calculate midplane ($x^* = 0$) and surface ($x^* = 1$) temperatures θ^* for $Fo = 0.1$ and 1, using $Bi = 0.1$, 1, and 10. Consider only the first four eigenvalues. Based on these results discuss the validity of the approximate solutions, Equations 5.40 and 5.41.

5.31 Consider the one-dimensional wall shown in the sketch, which is initially at a uniform temperature T_i and is suddenly subjected to the convection boundary condition with a fluid at T_∞.

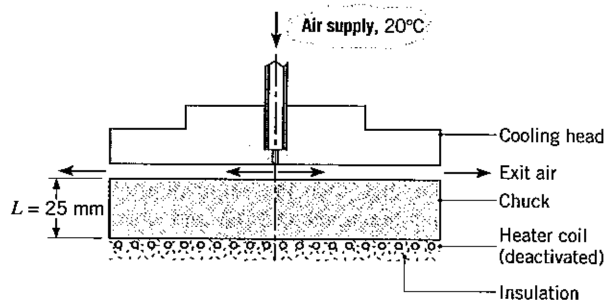


For a particular wall, case 1, the temperature at $x = L_1$ after $t_1 = 100$ s is $T_1(L_1, t_1) = 315°\text{C}$. Another wall, case 2, has different thickness and thermal conditions as shown below.

Case	L (m)	α (m²/s)	k (W/m · K)	T_i (°C)	T_∞ (°C)	h (W/m² · K)
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

How long will it take for the second wall to reach 28.5°C at the position $x = L_2$? Use as the basis for analysis, the dimensionless functional dependence for the transient temperature distribution expressed in Equation 5.38.

5.32 Referring to the semiconductor processing tool of Problem 5.12, it is desired at some point in the manufacturing cycle to cool the chuck, which is made of aluminum alloy 2024. The proposed cooling scheme passes air at 15°C between the air-supply head and the chuck surface.



(a) If the chuck is initially at a uniform temperature of 100°C, calculate the time required for its lower surface to reach 25°C, assuming a uniform convection coefficient of $50\ \text{W/m}^2\cdot\text{K}$ at the head–chuck interface.

$\boxed{\text{(b)}}$ Generate a plot of the time-to-cool as a function of the convection coefficient for the range $10 \le h \le 2000\ \text{W/m}^2\cdot\text{K}$. If the lower limit represents a free convection condition without any head present, comment on the effectiveness of the head design as a method for cooling the chuck.

5.33 Consider the thermal energy storage unit of Problem 5.10, but with a masonry material of $\rho = 1900\ \text{kg/m}^3$, $c = 800\ \text{J/kg}\cdot\text{K}$, and $k = 0.70\ \text{W/m}\cdot\text{K}$ used in place of the aluminum. How long will it take to achieve 75% of the maximum possible energy storage? What are the maximum and minimum temperatures of the masonry at this time?

5.34 Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 100-mm-thick steel plate ($\rho = 7830\ \text{kg/m}^3$, $c = 550\ \text{J/kg}\cdot\text{K}$, $k = 48\ \text{W/m}\cdot\text{K}$), which is initially at a uniform temperature of $T_i = 200°\text{C}$ and is to be heated to a minimum temperature of 550°C. Heating is effected in a gas-fired furnace, where products of combustion at $T_\infty = 800°\text{C}$ maintain a convection coefficient of $h = 250\ \text{W/m}^2\cdot\text{K}$ on both surfaces of the plate. How long should the plate be left in the furnace?

$\boxed{\text{5.35}}$ Consider the heavily insulated pipe of Example 5.4, which is suddenly subjected to the flow of hot oil. Use the *Transient Conduction*, *Plane Wall* model of *IHT* to obtain the following solutions.

(a) Calculate the temperature of the inner and outer surfaces of the pipe, the heat flux at the inner surface, and the energy transferred to the wall after 8 min. Compare your results with those obtained in the example.

(b) At what time will the outer surface temperature of the pipe, $T(0, t)$, reach 25°C?

(c) Using the *Explore* and *Graph* options of *IHT*, calculate and plot on a single graph the temperature distributions, $T(x, t)$, for the initial condition, the final condition, and intermediate times of 4 and 8 min. Explain key features of the distributions.

(d) Calculate and plot the temperature histories, $T(x, t)$, at the inner ($x = 0$) and outer ($x = L$) pipe surfaces for $0 \leq t \leq 16$ min.

5.36 The 150-mm-thick wall of a gas-fired furnace is constructed of fire-clay brick ($k = 1.5\ \mathrm{W/m \cdot K}$, $\rho = 2600\ \mathrm{kg/m^3}$, $c_p = 1000\ \mathrm{J/kg \cdot K}$) and is well insulated at its outer surface. The wall is at a uniform initial temperature of 20°C, when the burners are fired and the inner surface is exposed to products of combustion for which $T_\infty = 950°\mathrm{C}$ and $h = 100\ \mathrm{W/m^2 \cdot K}$.

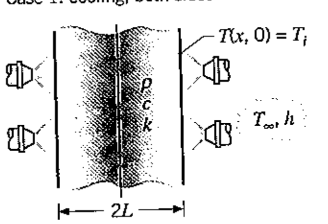
(a) How long does it take for the outer surface of the wall to reach a temperature of 750°C?

(b) Plot the temperature distribution in the wall at the foregoing time, as well as at several intermediate times.

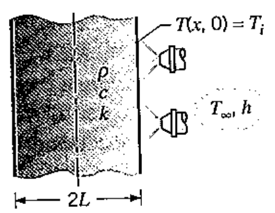
5.37 Steel is sequentially heated and cooled (*annealed*) to relieve stresses and to make it less brittle. Consider a 100-mm-thick plate ($k = 45\ \mathrm{W/m \cdot K}$, $\rho = 7800\ \mathrm{kg/m^3}$, $c_p = 500\ \mathrm{J/kg \cdot K}$) that is initially at a uniform temperature of 300°C and is heated (on both sides) in a gas-fired furnace for which $T_\infty = 700°\mathrm{C}$ and $h = 500\ \mathrm{W/m^2 \cdot K}$. How long will it take for a minimum temperature of 550°C to be reached in the plate?

5.38 A plate of thickness $2L = 25$ mm at a temperature of 600°C is removed from a hot pressing operation and must be cooled rapidly in order to achieve the required physical properties. The process engineer plans to use air jets to control the rate of cooling, but she is uncertain whether it is necessary to cool both sides (case 1) or only one side (case 2) of the plate. The concern is not just for the *time-to-cool*, but also for the maximum temperature difference within the plate. If this temperature difference is too large, the plate can experience significant warping.

Case 1: cooling, both sides



Case 2: cooling, one side only



The air supply is at 25°C, and the convection coefficient on the surface is $400\ \mathrm{W/m^2 \cdot K}$. The thermophysical properties of the plate are $\rho = 3000\ \mathrm{kg/m^3}$, $c = 750\ \mathrm{J/kg \cdot K}$, and $k = 15\ \mathrm{W/m \cdot K}$.

(a) Using the *IHT* software, calculate and plot on one graph the temperature histories for cases 1 and 2 for a 500-s cooling period. Compare the times required for the maximum temperature in the plate to reach 100°C. Assume no heat loss from the unexposed surface of case 2.

(b) For both cases, calculate and plot on one graph the variation with time of the maximum temperature difference in the plate. Comment on the relative magnitudes of the temperature gradients within the plate as a function of time.

5.39 During transient operation, the steel nozzle of a rocket engine must not exceed a maximum allowable operating temperature of 1500 K when exposed to combustion gases characterized by a temperature of 2300 K and a convection coefficient of $5000\ \mathrm{W/m^2 \cdot K}$. To extend the duration of engine operation, it is proposed that a ceramic *thermal barrier coating* ($k = 10\ \mathrm{W/m \cdot K}$, $\alpha = 6 \times 10^{-6}\ \mathrm{m^2/s}$) be applied to the interior surface of the nozzle.

(a) If the ceramic coating is 10 mm thick and at an initial temperature of 300 K, obtain a conservative estimate of the maximum allowable duration of engine operation. The nozzle radius is much larger than the combined wall and coating thickness.

(b) Compute and plot the inner and outer surface temperatures of the coating as a function of time for $0 \leq t \leq 150$ s. Repeat the calculations for a coating thickness of 40 mm.

5.40 In a tempering process, glass plate, which is initially at a uniform temperature T_i, is cooled by suddenly reducing the temperature of both surfaces to T_s. The plate is 20 mm thick, and the glass has a thermal diffusivity of $6 \times 10^{-7}\ \mathrm{m^2/s}$.

(a) How long will it take for the midplane temperature to achieve 50% of its maximum possible temperature reduction?

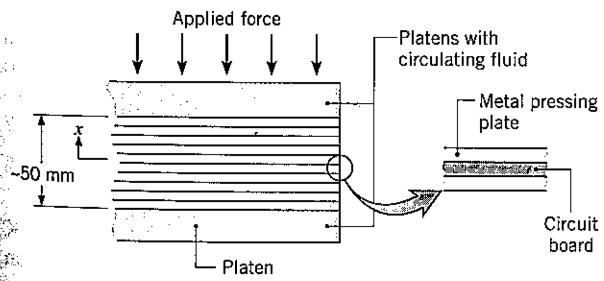
(b) If $(T_i - T_s) = 300°\mathrm{C}$, what is the maximum temperature gradient in the glass at the above time?

5.41 The strength and stability of tires may be enhanced by heating both sides of the rubber ($k = 0.14\ \mathrm{W/m \cdot K}$, $\alpha = 6.35 \times 10^{-8}\ \mathrm{m^2/s}$) in a steam chamber for which $T_\infty = 200°\mathrm{C}$. In the heating process, a 20-mm-thick rubber wall (assumed to be unthreaded) is taken from an initial temperature of 25°C to a midplane temperature of 150°C.

(a) If steam flow over the tire surfaces maintains a convection coefficient of $h = 200\ \mathrm{W/m^2 \cdot K}$, how long will it take to achieve the desired midplane temperature?

(b) To accelerate the heating process, it is recommended that the steam flow be made sufficiently vigorous to maintain the tire surfaces at 200°C throughout the process. Compute and plot the midplane and surface temperatures for this case, as well as for the conditions of part (a).

5.42 Copper-coated, epoxy-filled fiberglass circuit boards are treated by heating a stack of them under high pressure as shown in the sketch. The purpose of the pressing–heating operation is to cure the epoxy that bonds the fiberglass sheets, imparting stiffness to the boards. The stack, referred to as a *book*, is comprised of 10 *boards* and 11 *pressing plates*, which prevent epoxy from flowing between the boards and impart a smooth finish to the cured boards. In order to perform simplified thermal analyses, it is reasonable to approximate the book as having an effective thermal conductivity (k) and an effective thermal capacitance (ρc_p). Calculate the effective properties if each of the boards and plates has a thickness of 2.36 mm and the following thermophysical properties: board (b) $\rho_b = 1000\ \mathrm{kg/m^3}$, $c_{p,b} = 1500\ \mathrm{J/kg \cdot K}$, $k_b = 0.30\ \mathrm{W/m \cdot K}$; plate ($p$) $\rho_p = 8000\ \mathrm{kg/m^3}$, $c_{p,p} = 480\ \mathrm{J/kg \cdot K}$, $k_p = 12\ \mathrm{W/m \cdot K}$.



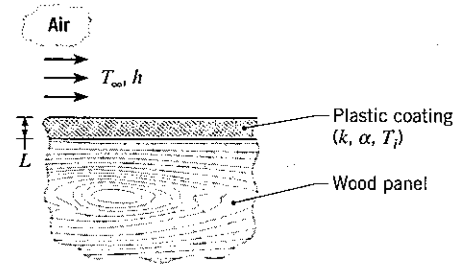
5.43 Circuit boards are treated by heating a stack of them under high pressure as illustrated in Problem 5.42. The platens at the top and bottom of the stack are maintained at a uniform temperature by a circulating fluid. The purpose of the pressing–heating operation is to cure the epoxy, which bonds the fiberglass sheets, and impart stiffness to the boards. The cure condition is achieved when the epoxy has been maintained at or above 170°C for at least 5 min. The effective thermophysical properties of the stack or *book* (boards and metal pressing plates) are $k = 0.613\ \mathrm{W/m \cdot K}$ and $\rho c_p = 2.73 \times 10^6\ \mathrm{J/m^3 \cdot K}$.

(a) If the book is initially at 15°C and, following application of pressure, the platens are suddenly brought to a uniform temperature of 190°C, calculate the elapsed time t_e required for the midplane of the book to reach the cure temperature of 170°C.

(b) If, at this instant of time, $t = t_e$, the platen temperature were reduced suddenly to 15°C, how much energy would have to be removed from the book by the coolant circulating in the platen, in order to return the stack to its initial uniform temperature?

5.44 An ice layer forms on a 5-mm-thick windshield of a car while parked during a cold night for which the ambient temperature is −20°C. Upon start-up, using a new defrost system, the interior surface is suddenly exposed to an airstream at 30°C. Assuming that the ice behaves as an insulating layer on the exterior surface, what interior convection coefficient would allow the exterior surface to reach 0°C in 60 s? The windshield thermophysical properties are $\rho = 2200\ \mathrm{kg/m^3}$, $c_p = 830\ \mathrm{J/kg \cdot K}$, and $k = 1.2\ \mathrm{W/m \cdot K}$.

5.45 A plastic coating is applied to wood panels by first depositing molten polymer on a panel and then cooling the surface of the polymer by subjecting it to air flow at 25°C. As first approximations, the heat of reaction associated with solidification of the polymer may be neglected and the polymer/wood interface may be assumed to be adiabatic.



If the thickness of the coating is $L = 2$ mm and it has an initial uniform temperature of $T_i = 200°\mathrm{C}$, how long will it take for the surface to achieve a *safe-to-touch* temperature of 42°C if the convection coefficient is $h = 200\ \mathrm{W/m^2 \cdot K}$? What is the corresponding value of the interface temperature? The thermal conductivity and diffusivity of the plastic are $k = 0.25\ \mathrm{W/m \cdot K}$ and $\alpha = 1.20 \times 10^{-7}\ \mathrm{m^2/s}$, respectively.

One-Dimensional Conduction: The Long Cylinder

5.46 Cylindrical steel rods (AISI 1010), 50 mm in diameter, are heat treated by drawing them through an oven 5 m long in which air is maintained at 750°C. The rods enter at 50°C and achieve a centerline temperature of 600°C before leaving. For a convection coefficient of 125 W/m^2 · K, estimate the speed at which the rods must be drawn through the oven.

5.47 Estimate the time required to cook a hot dog in boiling water. Assume that the hot dog is initially at 6°C, that the convection heat transfer coefficient is 100 W/m^2 · K, and that the final temperature is 80°C at the centerline. Treat the hot dog as a long cylinder of 20-mm diameter having the properties: ρ = 880 kg/m^3, c = 3350 J/kg · K, and k = 0.52 W/m · K.

5.48 A long rod of 60-mm diameter and thermophysical properties ρ = 8000 kg/m^3, c = 500 J/kg · K, and k = 50 W/m · K is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/m^2 · K.

(a) What is the centerline temperature of the rod when the surface temperature is 550 K?

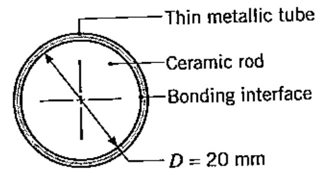
(b) In a heat-treating process, the centerline temperature of the rod must be increased from T_i = 300 K to T = 500 K. Compute and plot the centerline temperature histories for h = 100, 500, and 1000 W/m^2 · K. In each case the calculation may be terminated when T = 500 K.

5.49 A long cylinder of 30-mm diameter, initially at a uniform temperature of 1000 K, is suddenly quenched in a large, constant-temperature oil bath at 350 K. The cylinder properties are k = 1.7 W/m · K, c = 1600 J/kg · K, and ρ = 400 kg/m^3, while the convection coefficient is 50 W/m^2 · K.

(a) Calculate the time required for the surface of the cylinder to reach 500 K.

(b) Compute and plot the surface temperature history for $0 \le t \le 300$ s. If the oil were agitated, providing a convection coefficient of 250 W/m^2 · K, how would the temperature history change?

5.50 A long pyroceram rod of diameter 20 mm is clad with a very thin metallic tube for mechanical protection. The bonding between the rod and the tube has a thermal contact resistance of $R'_{t,c}$ = 0.12 m · K/W.



(a) If the rod is initially at a uniform temperature of 900 K and is suddenly cooled by exposure to an airstream for which T_∞ = 300 K and h = 100 W/m^2 · K, at which time will the centerline reach 600 K?

(b) Cooling may be accelerated by increasing the airspeed and hence the convection coefficient. For values of h = 100, 500, and 1000 W/m^2 · K, compute and plot the centerline and surface temperatures of the pyroceram as a function of time for $0 \le t \le 300$ s. Comment on the implications of achieving enhanced cooling solely by increasing h.

5.51 A long rod 40 mm in diameter, fabricated from sapphire (aluminum oxide) and initially at a uniform temperature of 800 K, is suddenly cooled by a fluid at 300 K having a heat transfer coefficient of 1600 W/m^2 · K. After 35 s, the rod is wrapped in insulation and experiences no heat losses. What will be the temperature of the rod after a long period of time?

5.52 A long bar of 70-mm diameter and initially at 90°C is cooled by immersing it in a water bath that is at 40°C and provides a convection coefficient of 20 W/m^2 · K. The thermophysical properties of the bar are ρ = 2600 kg/m^3, c = 1030 J/kg · K, and k = 3.50 W/m · K.

(a) How long should the bar remain in the bath in order that, when it is removed and allowed to equilibrate while isolated from any surroundings, it achieves a uniform temperature of 55°C?

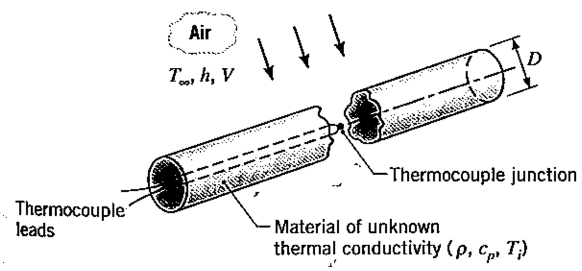
(b) What is the surface temperature of the bar when it is removed from the bath?

5.53 A long plastic rod of 30-mm diameter (k = 0.3 W/m · K and ρc_p = 1040 kJ/m^3 · K) is uniformly heated in an oven as preparation for a pressing operation. For best results, the temperature in the rod should not be less than 200°C. To what uniform temperature should the rod be heated in the oven if, for the worst case, the rod sits on a conveyor for 3 min while exposed to convection cooling with ambient air at 25°C and with a convection coefficient of 8 W/m^2 · K? A further condition for good results is a maximum–minimum temperature difference of less than 10°C. Is this condition satisfied and, if not, what could you do to satisfy it?

5.54 As part of a heat treatment process, cylindrical, 304 stainless steel rods of 100-mm diameter are cooled from an initial temperature of 500°C by suspending them in an oil bath at 30°C. If a convection coefficient of 500 W/m^2 · K is maintained by circulation of the oil, how long does it take for the centerline of a rod to reach a temperature of 50°C, at which point it is withdrawn from the bath? If 10 rods of length L = 1 m are processed per hour, what is the nominal rate at which energy must be extracted from the bath (the cooling load)?

5.55 In a manufacturing process, long rods of different diameters are at a uniform temperature of 400°C in a curing oven, from which they are removed and cooled by forced convection in air at 25°C. One of the line operators has observed that it takes 280 seconds for a 40-mm-diameter rod to cool to a *safe-to-handle* temperature of 60°C. For an equivalent convection coefficient, how long will it take for a 80-mm-diameter rod to cool to the same temperature? The thermophysical properties of the rod are ρ = 2500 kg/m^3, c = 900 J/kg · K, and k = 15 W/m · K. Comment on your result. Did you anticipate this outcome?

5.56 The density and specific heat of a particular material are known (ρ = 1200 kg/m^3, c_p = 1250 J/kg · K), but its thermal conductivity is unknown. To determine the thermal conductivity, a long cylindrical specimen of diameter D = 40 mm is machined, and a thermocouple is inserted through a small hole drilled along the centerline.



The thermal conductivity is determined by performing an experiment in which the specimen is heated to a uniform temperature of T_i = 100°C and then cooled by passing air at T_∞ = 25°C in cross flow over the cylinder. For the prescribed air velocity, the convection coefficient is h = 55 W/m^2 · K.

(a) If a centerline temperature of $T(0, t)$ = 40°C is recorded after t = 1136 s of cooling, verify that the material has a thermal conductivity of k = 0.30 W/m · K.

(b) For air in cross-flow over the cylinder, the prescribed value of h = 55 W/m^2 · K corresponds to a velocity of V = 6.8 m/s. If $h = CV^{0.618}$, where the constant C has units of W · s$^{0.618}$/m$^{2.618}$ · K, how does the centerline temperature at t = 1136 s vary with velocity for $3 \le V \le 20$ m/s? Determine the centerline temperature histories for $0 \le t \le 1500$ s and velocities of 3, 10, and 20 m/s.

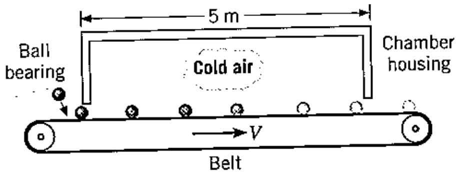
5.57 In Section 5.2 we noted that the value of the Biot number significantly influences the nature of the temperature distribution in a solid during a transient conduction process. Reinforce your understanding of this important concept by using the *IHT* model for one-dimensional transient conduction to determine radial temperature distributions in a 30-mm-diameter, stainless steel rod (k = 15 W/m · K, ρ = 8000 kg/m^3, c_p = 475 J/kg · K), as it is cooled from an initial uniform temperature of 325°C by a fluid at 25°C. For the following values of the convection coefficient and the designated times, determine the radial temperature distribution: h = 100 W/m^2 · K (t = 0, 100, 500 s); h = 1000 W/m^2 · K (t = 0, 10, 50 s); h = 5000 W/m^2 · K (t = 0, 1, 5, 25 s). Prepare a separate graph for each convection coefficient, on which temperature is plotted as a function of dimensionless radius at the designated times.

One-Dimensional Conduction: The Sphere

5.58 In heat treating to harden steel ball bearings (c = 500 J/kg · K, ρ = 7800 kg/m^3, k = 50 W/m · K), it is desirable to increase the surface temperature for a short time without significantly warming the interior of the ball. This type of heating can be accomplished by sudden immersion of the ball in a molten salt bath with T_∞ = 1300 K and h = 5000 W/m^2 · K. Assume that any location within the ball whose temperature exceeds 1000 K will be hardened. Estimate the time required to harden the outer millimeter of a ball of diameter 20 mm, if its initial temperature is 300 K.

5.59 A sphere of 80-mm diameter (k = 50 W/m · K and α = 1.5 × 10^{-6} m^2/s) is initially at a uniform, elevated temperature and is quenched in an oil bath maintained at 50°C. The convection coefficient for the cooling process is 1000 W/m^2 · K. At a certain time, the surface temperature of the sphere is measured to be 150°C. What is the corresponding center temperature of the sphere?

5.60 A cold air chamber is proposed for quenching steel ball bearings of diameter $D = 0.2$ m and initial temperature $T_i = 400°C$. Air in the chamber is maintained at $-15°C$ by a refrigeration system, and the steel balls pass through the chamber on a conveyor belt. Optimum bearing production requires that 70% of the initial thermal energy content of the ball above $-15°C$ be removed. Radiation effects may be neglected, and the convection heat transfer coefficient within the chamber is 1000 W/m$^2 \cdot$ K. Estimate the residence time of the balls within the chamber, and recommend a drive velocity of the conveyor. The following properties may be used for the steel: $k = 50$ W/m $\cdot$ K, $\alpha = 2 \times 10^{-5}$ m^2/s, and $c = 450$ J/kg $\cdot$ K.



5.61 Stainless steel (AISI 304) ball bearings, which have uniformly been heated to 850°C, are hardened by quenching them in an oil bath that is maintained at 40°C. The ball diameter is 20 mm, and the convection coefficient associated with the oil bath is 1000 W/m$^2 \cdot$ K.

(a) If quenching is to occur until the surface temperature of the balls reaches 100°C, how long must the balls be kept in the oil? What is the center temperature at the conclusion of the cooling period?

(b) If 10,000 balls are to be quenched per hour, what is the rate at which energy must be removed by the oil bath cooling system in order to maintain its temperature at 40°C?

5.62 A spherical hailstone that is 5 mm in diameter is formed in a high-altitude cloud at $-30°C$. If the stone begins to fall through warmer air at 5°C, how long will it take before the outer surface begins to melt? What is the temperature of the stone's center at this point in time, and how much energy (J) has been transferred to the stone? A convection heat transfer coefficient of 250 W/m$^2 \cdot$ K may be assumed, and the properties of the hailstone may be taken to be those of ice.

5.63 A sphere 30 mm in diameter initially at 800 K is quenched in a large bath having a constant temperature of 320 K with a convection heat transfer coefficient of 75 W/m$^2 \cdot$ K. The thermophysical properties of the sphere material are: $\rho = 400$ kg/m^3, $c = 1600$ J/kg $\cdot$ K, and $k = 1.7$ W/m $\cdot$ K.

(a) Show, in a qualitative manner on T–t coordinates, the temperatures at the center and at the surface of the sphere as a function of time.

(b) Calculate the time required for the surface of the sphere to reach 415 K.

(c) Determine the heat flux (W/m^2) at the outer surface of the sphere at the time determined in part (b).

(d) Determine the energy (J) that has been lost by the sphere during the process of cooling to the surface temperature of 415 K.

(e) At the time determined by part (b), the sphere is quickly removed from the bath and covered with perfect insulation, such that there is no heat loss from the surface of the sphere. What will be the temperature of the sphere after a long period of time has elapsed?

$\boxed{\text{(f)}}$ Compute and plot the center and surface temperature histories over the period $0 \leq t \leq 150$ s. What effect does an increase in the convection coefficient to $h = 200$ W/m$^2 \cdot$ K have on the foregoing temperature histories? For $h = 75$ and 200 W/m$^2 \cdot$ K, compute and plot the surface heat flux as a function of time for $0 \leq t \leq 150$ s.

5.64 Spheres A and B are initially at 800 K, and they are simultaneously quenched in large constant temperature baths, each having a temperature of 320 K. The following parameters are associated with each of the spheres and their cooling processes.

	Sphere A	Sphere B
Diameter (mm)	300	30
Density (kg/m^3)	1600	400
Specific heat (kJ/kg $\cdot$ K)	0.400	1.60
Thermal conductivity (W/m $\cdot$ K)	170	1.70
Convection coefficient (W/m$^2 \cdot$ K)	5	50

(a) Show in a qualitative manner, on T versus t coordinates, the temperatures at the center and at the surface for each sphere as a function of time. Briefly explain the reasoning by which you determine the relative positions of the curves.

(b) Calculate the time required for the surface of each sphere to reach 415 K.

(c) Determine the energy that has been gained by each of the baths during the process of the spheres cooling to 415 K.

5.65 Spheres of 40-mm diameter heated to a uniform temperature of 400°C are suddenly removed from the oven and placed in a forced-air bath operating at 25°C with a convection coefficient of 300 W/m$^2 \cdot$ K on the sphere surfaces. The thermophysical properties of the sphere material are $\rho = 3000$ kg/m^3, $c = 850$ J/kg $\cdot$ K, and $k = 15$ W/m $\cdot$ K.

(a) How long must the spheres remain in the air bath for 80% of the thermal energy to be removed?

(b) The spheres are then placed in a packing carton that prevents further heat transfer to the environment. What uniform temperature will the spheres eventually reach?

5.66 Consider the packed bed operating conditions of Problem 5.11, but with Pyrex ($\rho = 2225$ kg/m^3, $c = 835$ J/kg $\cdot$ K, $k = 1.4$ W/m $\cdot$ K) used instead of aluminum. How long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere?

5.67 The convection coefficient for flow over a solid sphere may be determined by submerging the sphere, which is initially at 25°C, into the flow, which is at 75°C, and measuring its surface temperature at some time during the transient heating process.

(a) If the sphere has a diameter of 0.1 m, a thermal conductivity of 15 W/m $\cdot$ K, and a thermal diffusivity of 10^{-5} m^2/s, at what time will a surface temperature of 60°C be recorded if the convection coefficient is 300 W/m$^2 \cdot$ K?

$\boxed{\text{(b)}}$ Assess the effect of thermal diffusivity on the thermal response of the material by computing center and surface temperature histories for $\alpha = 10^{-6}$, 10^{-5}, and 10^{-4} m^2/s. Plot your results for the period $0 \leq t \leq 300$ s. In a similar manner, assess the effect of thermal conductivity by considering values of $k = 1.5$, 15, and 150 W/m $\cdot$ K.

5.68 In a process to manufacture glass beads ($k = 1.4$ W/m $\cdot$ K, $\rho = 2200$ kg/m^3, $c_p = 800$ J/kg $\cdot$ K) of 3-mm diameter, the beads are suspended in an upwardly directed airstream that is at $T_\infty = 15°C$ and maintains a convection coefficient of $h = 400$ W/m$^2 \cdot$ K.

(a) If the beads are at an initial temperature of $T_i = 477°C$, how long must they be suspended to achieve a center temperature of 80°C? What is the corresponding surface temperature?

$\boxed{\text{(b)}}$ Compute and plot the center and surface temperatures as a function of time for $0 \leq t \leq 20$ s and $h = 100$, 400, and 1000 W/m$^2 \cdot$ K.

$\boxed{\textbf{5.69}}$ Consider the sphere of Example 5.5, which is initially at a uniform temperature when it is suddenly removed from the furnace and subjected to a two-step cooling process. Use the *Transient Conduction*, *Sphere* model of *IHT* to obtain the following solutions.

(a) For step 1, calculate the time required for the center temperature to reach $T(0, t) = 335°C$, while cooling in air at 20°C with a convection coefficient of 10 W/m$^2 \cdot$ K. What is the Biot number for this cooling process? Do you expect radial temperature gradients to be appreciable? Compare your results to those of the example.

(b) For step 2, calculate the time required for the center temperature to reach $T(0, t) = 50°C$, while cooling in a water bath at 20°C with a convection coefficient of 6000 W/m$^2 \cdot$ K.

(c) For the step 2 cooling process, calculate and plot the temperature histories, $T(r, t)$, for the center and surface of the sphere. Identify and explain key features of the histories. When do you expect the temperature gradients in the sphere to be the largest?

Semi-infinite Media

5.70 Two large blocks of different materials, such as copper and concrete, have been sitting in a room (23°C) for a very long time. Which of the two blocks, if either, will feel colder to the touch? Assume the blocks to be semi-infinite solids and your hand to be at a temperature of 37°C.

5.71 Asphalt pavement may achieve temperatures as high as 50°C on a hot summer day. Assume that such a temperature exists throughout the pavement, when suddenly a rainstorm reduces the surface temperature to 20°C. Calculate the total amount of energy (J/m^2) that will be transferred from the asphalt over a 30-min period in which the surface is maintained at 20°C.

5.72 A thick steel slab ($\rho = 7800$ kg/m^3, $c = 480$ J/kg $\cdot$ K, $k = 50$ W/m $\cdot$ K) is initially at 300°C and is cooled by water jets impinging on one of its surfaces. The temperature of the water is 25°C, and the jets maintain an extremely large, approximately uniform convection coefficient at the surface. Assuming that the surface is maintained at the temperature of the water throughout the cooling, how long will it take for the temperature to reach 50°C at a distance of 25 mm from the surface?

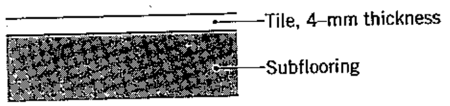
.73 Consider the water main of Example 5.6, which is buried in soil initially at 20°C and is suddenly subjected to a constant surface temperature of −15°C for 60 days. Use the *Transient Conduction/Semi-Infinite Solid* model of *IHT* to obtain the following solutions. Compare your results with those in the Comments section of the example.

(a) Calculate and plot the temperature history at the burial depth of 0.68 m for thermal diffusivities of a $\alpha \times 10^7 = 1.0, 1.38$, and $3.0\ \mathrm{m^2/s}$.

(b) For $\alpha = 1.38 \times 10^{-7}\ \mathrm{m^2/s}$, plot the temperature distribution over the depth $0 \leq x \leq 1.0$ m for times of 1, 5, 10, 30, and 60 days.

(c) For $\alpha = 1.38 \times 10^{-7}\ \mathrm{m^2/s}$, show that the heat flux from the soil decreases with increasing time by plotting $q''_x(0, t)$ as a function of time for the 60-day period. On this graph, also plot the heat flux at the depth of the buried main, $q''_x(0.68\ m, t)$.

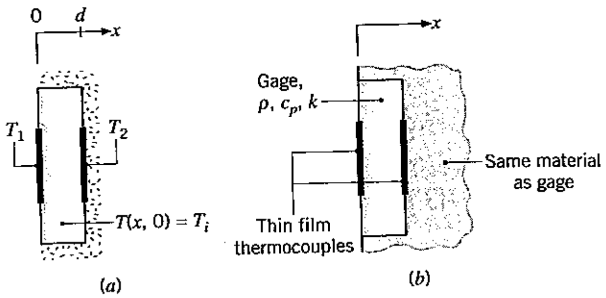
5.74 A tile-iron consists of a massive plate maintained at 150°C by an imbedded electrical heater. The iron is placed in contact with a tile to soften the adhesive, allowing the tile to be easily lifted from the subflooring. The adhesive will soften sufficiently if heated above 50°C for at least 2 min, but its temperature should not exceed 120°C to avoid deterioration of the adhesive. Assume the tile and subfloor to have an initial temperature of 25°C and to have equivalent thermophysical properties of $k = 0.15$ W/m · K and $\rho c_p = 1.5 \times 10^6\ \mathrm{J/m^3 \cdot K}$.



(a) How long will it take a worker using the tile-iron to lift a tile? Will the adhesive temperature exceed 120°C?

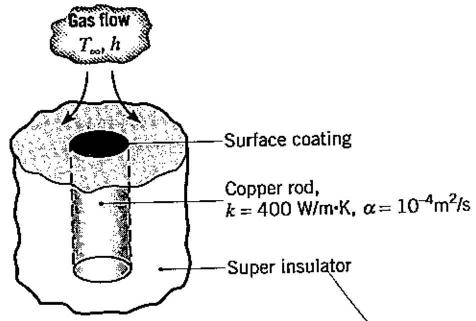
(b) If the tile-iron has a square surface area 254 mm to the side, how much energy has been removed from it during the time it has taken to lift the tile?

5.75 The manufacturer of a heat flux gage like that illustrated in Problem 1.12 claims the time constant for a 63.2% response to be $\tau = (4d^2\rho c_p)/\pi^2 k$, where ρ, c_p, and k are the thermophysical properties of the gage material and d is its thickness. Not knowing the origin of this relation, your task is to model the gage considering the two extreme cases illustrated below. In both cases, the gage, initially at a uniform temperature T_i, is exposed to a sudden change in surface temperature, $T(0, t) = T_s$. For case (*a*) the backside of the gage is insulated, and for case (*b*) the gage is imbedded in a semi-infinite solid having the same thermophysical properties as those of the gage.



Develop relationships for predicting the time constant of the gage for the two cases and compare them to the manufacturer's relation. What conclusion can you draw from this analysis regarding the transient response of gages for different applications?

5.76 A simple procedure for measuring surface convection heat transfer coefficients involves coating the surface with a thin layer of material having a precise melting point temperature. The surface is then heated and, by determining the time required for melting to occur, the convection coefficient is determined. The following experimental arrangement uses the procedure to determine the convection coefficient for gas flow normal to a surface. Specifically, a long copper rod is encased in a super insulator of very low thermal conductivity, and a very thin coating is applied to its exposed surface.



If the rod is initially at 25°C and gas flow for which $h = 200\ \mathrm{W/m^2 \cdot K}$ and $T_\infty = 300$°C is initiated, what is the melting point temperature of the coating if melting is observed to occur at $t = 400$ s?

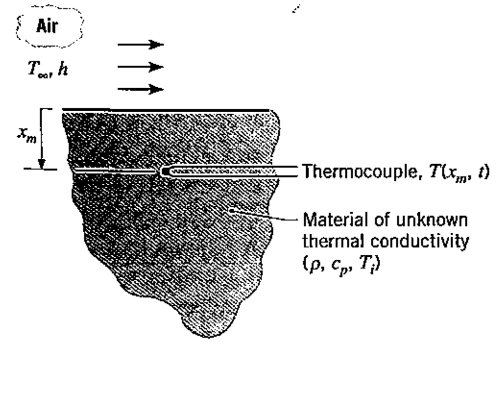
$\boxed{5.77}$ An insurance company has hired you as a consultant to improve their understanding of burn injuries. They are especially interested in injuries induced when a portion of a worker's body comes into contact with machinery that is at elevated temperatures in the range of 50 to 100°C. Their medical consultant informs them that irreversible thermal injury (cell death) will occur in any living tissue that is maintained at $T \geq 48$°C for a duration $\Delta t \geq 10$ s. They want information concerning the extent of irreversible tissue damage (as measured by distance from the skin surface) as a function of the machinery temperature and the time during which contact is made between the skin and the machinery. Assume that living tissue has a normal temperature of 37°C, is isotropic, and has constant properties equivalent to those of liquid water.

(a) To assess the seriousness of the problem, compute locations in the tissue at which the temperature will reach 48°C after 10 s of exposure to machinery at 50°C and 100°C.

(b) For a machinery temperature of 100°C and $0 \leq t \leq 30$ s, compute and plot temperature histories at tissue locations of 0.5, 1 and 2 mm from the skin.

5.78 A procedure for determining the thermal conductivity of a solid material involves embedding a thermocouple in a thick slab of the solid and measuring the response to a prescribed change in temperature at one surface. Consider an arrangement for which the thermocouple is embedded 10 mm from a surface that is suddenly brought to a temperature of 100°C by exposure to boiling water. If the initial temperature of the slab was 30°C and the thermocouple measures a temperature of 65°C, 2 min after the surface is brought to 100°C, what is its thermal conductivity? The density and specific heat of the solid are known to be 2200 kg/m³ and 700 J/kg · K.

5.79 The density and specific heat of a plastic material are known ($\rho = 950\ \mathrm{kg/m^3}$, $c_p = 1100$ J/kg · K), but its thermal conductivity is unknown. To determine the thermal conductivity, an experiment is performed in which a thick sample of the material is heated to a uniform temperature of 100°C and then cooled by passing air at 25°C over one surface. A thermocouple embedded a distance of $x_m = 10$ mm below the surface records the thermal response of the plastic during cooling.



If the convection coefficient associated with the air flow is $h = 200\ \mathrm{W/m^2 \cdot K}$ and a temperature of 60°C is recorded 5 min after the onset of cooling, what is the thermal conductivity of the material?

5.80 A very thick slab with thermal diffusivity $5.6 \times 10^{-6}\ \mathrm{m^2/s}$ and thermal conductivity 20 W/m · K is initially at a uniform temperature of 325°C. Suddenly, the surface is exposed to a coolant at 15°C for which the convection heat transfer coefficient is 100 $\mathrm{W/m^2 \cdot K}$.

(a) Determine temperatures at the surface and at a depth of 45 mm after 3 min have elapsed.

$\boxed{(b)}$ Compute and plot temperature histories ($0 \leq t \leq 300$ s) at $x = 0$ and $x = 45$ mm for the following parametric variations: (i) $\alpha = 5.6 \times 10^{-7}$, 5.6×10^{-6}, and $5.6 \times 10^{-5}\ \mathrm{m^2/s}$; and (ii) $k = 2, 20$, and 200 W/m · K.

5.81 A thick oak wall, initially at 25°C, is suddenly exposed to combustion products for which $T_\infty = 800$°C and $h = 20\ \mathrm{W/m^2 \cdot K}$.

(a) Determine the time of exposure required for the surface to reach the ignition temperature of 400°C.

$\boxed{(b)}$ Plot the temperature distribution $T(x)$ in the medium at $t = 325$ s. The distribution should extend to a location for which $T \approx 25$°C.

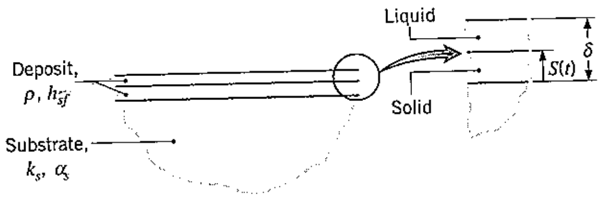
5.82 Standards for firewalls may be based on their thermal response to a prescribed radiant heat flux. Consider a 0.25-m-thick concrete wall ($\rho = 2300\ \mathrm{kg/m^3}$, $c = 880$ J/kg · K, $k = 1.4$ W/m · K), which is at an initial temperature of $T_i = 25$°C and irradiated at one surface by lamps that provide a uniform heat flux of $q''_s = 10^4\ \mathrm{W/m^2}$. The absorptivity of the surface to the irradiation is $\alpha_s = 1.0$. If building code requirements dictate that the temperatures of the irradiated and back surfaces must not exceed 325°C and 25°C, respectively, after 30 min of heating, will the requirements be met?

5.83 It is well known that, although two materials are at the same temperature, one may feel cooler to the touch than the other. Consider thick plates of copper and glass, each at an initial temperature of 300 K. Assuming your finger to be at an initial temperature of 310 K and to have thermophysical properties of

$\rho = 1000\ \text{kg/m}^3$, $c = 4180\ \text{J/kg}\cdot\text{K}$, and $k = 0.625\ \text{W/m}\cdot\text{K}$, determine whether the copper or the glass will feel cooler to the touch.

5.84 Two stainless steel plates ($\rho = 8000\ \text{kg/m}^3$, $c = 500\ \text{J/kg}\cdot\text{K}$, $k = 15\ \text{W/m}\cdot\text{K}$), each 20 mm thick and insulated on one surface, are initially at 400 and 300 K when they are pressed together at their uninsulated surfaces. What is the temperature of the insulated surface of the hot plate after 1 min has elapsed?

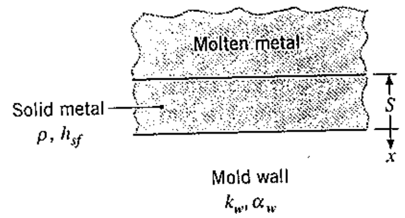
5.85 Special coatings are often formed by depositing thin layers of a molten material on a solid substrate. Solidification begins at the substrate surface and proceeds until the thickness S of the solid layer becomes equal to the thickness δ of the deposit.



(a) Consider conditions for which molten material at its fusion temperature T_f is deposited on a *large* substrate that is at an initial uniform temperature T_i. With $S = 0$ at $t = 0$, develop an expression for estimating the time t_d required to completely solidify the deposit if it remains at T_f throughout the solidification process. Express your result in terms of the substrate thermal conductivity and thermal diffusivity (k_s, α_s), the density and latent heat of fusion of the deposit (ρ, h_{sf}), the deposit thickness δ, and the relevant temperatures (T_f, T_i).

(b) The plasma spray deposition process of Problem 5.24 is used to apply a thin ($\delta = 2$ mm) alumina coating on a thick tungsten substrate. The substrate has a uniform initial temperature of $T_i = 300$ K, and its thermal conductivity and thermal diffusivity may be approximated as $k_s = 120\ \text{W/m}\cdot\text{K}$ and $\alpha_s = 4.0 \times 10^{-5}\ \text{m}^2/\text{s}$, respectively. The density and latent heat of fusion of the alumina are $\rho = 3970\ \text{kg/m}^3$ and $h_{sf} = 3577\ \text{kJ/kg}$, respectively, and the alumina solidifies at its fusion temperature ($T_f = 2318$ K). Assuming that the molten layer is instantaneously deposited on the substrate, estimate the time required for the deposit to solidify.

5.86 When a molten metal is cast in a mold that is a poor conductor, the dominant resistance to heat flow is within the mold wall. Consider conditions for which a liquid metal is solidifying in a thick-walled mold of thermal conductivity k_w and thermal diffusivity α_w. The density and latent heat of fusion of the metal are designated as ρ and h_{sf}, respectively, and in both its molten and solid states, the thermal conductivity of the metal is very much larger than that of the mold.



Just before the start of solidification ($S = 0$), the mold wall is everywhere at an initial uniform temperature T_i and the molten metal is everywhere at its fusion (melting point) temperature of T_f. Following the start of solidification, there is conduction heat transfer into the mold wall and the thickness of the solidified metal, S, increases with time t.

(a) Sketch the one-dimensional temperature distribution, $T(x)$, in the mold wall and the metal at $t = 0$ and at two subsequent times during the solidification. Clearly indicate any underlying assumptions.

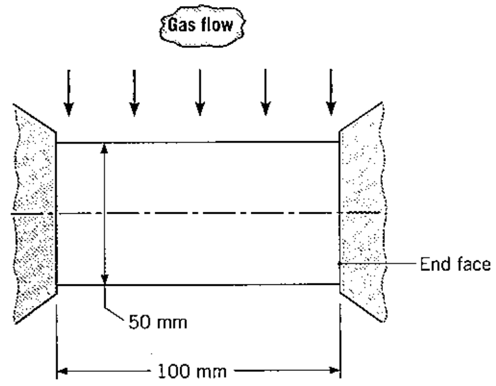
(b) Obtain a relation for the variation of the solid layer thickness S with time t, expressing your result in terms of appropriate parameters of the system.

Multidimensional Conduction

5.87 A long steel (plain carbon) billet of square cross section 0.3 m by 0.3 m, initially at a uniform temperature of 30°C, is placed in a soaking oven having a temperature of 750°C. If the convection heat transfer coefficient for the heating process is $100\ \text{W/m}^2\cdot\text{K}$, how long must the billet remain in the oven before its center temperature reaches 600°C?

5.88 Fireclay brick of dimensions 0.06 m × 0.09 m × 0.20 m is removed from a kiln at 1600 K and cooled in air at 40°C with $h = 50\ \text{W/m}^2\cdot\text{K}$. What is the temperature at the center and at the corners of the brick after 50 min of cooling?

5.89 A cylindrical copper pin 100 mm long and 50 mm in diameter is initially at a uniform temperature of 20°C. The end faces are suddenly subjected to an intense heating rate that raises them to a temperature of 500°C. At the same time, the cylindrical surface is subjected to heating by gas flow with a temperature of 500°C and a heat transfer coefficient of $100\ \text{W/m}^2\cdot\text{K}$.



(a) Determine the temperature at the center point of the cylinder 8 s after sudden application of the heat.

(b) Considering the parameters governing the temperature distribution in transient heat diffusion problems, can any simplifying assumptions be justified in analyzing this particular problem? Explain briefly.

5.90 Recalling that your mother once said that meat should be cooked until every portion has attained a temperature of 80°C, how long will it take to cook a 2.25-kg roast? Assume that the meat is initially at 6°C and that the oven temperature is 175°C with a convection heat transfer coefficient of $15\ \text{W/m}^2\cdot\text{K}$. Treat the roast as a cylinder with properties of liquid water, having a diameter equal to its length.

5.91 A long rod 20 mm in diameter is fabricated from alumina (polycrystalline aluminum oxide) and is initially at a uniform temperature of 850 K. The rod is suddenly exposed to fluid at 350 K with $h = 500\ \text{W/m}^2\cdot\text{K}$. Estimate the centerline temperature of the rod after 30 s at an exposed end and at an axial distance of 6 mm from the end.

$\boxed{5.92}$ Consider the stainless steel cylinder of Example 5.7, which is initially at 600 K and suddenly quenched in an oil bath at 300 K with $h = 500\ \text{W/m}^2\cdot\text{K}$. Use the *Transient Conduction, Plane Wall and Cylinder* models of *IHT* to obtain the following solutions.

(a) Calculate the temperatures, $T(r, x, t)$, after 3 min at the cylinder center, $T(0, 0, 3\ \text{min})$, at the center of a circular face, $T(0, L, 3\ \text{min})$, and at the midheight of the side, $T(r_o, 0, 3\ \text{min})$. Compare your results with those in the example.

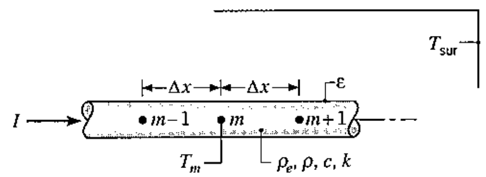
(b) Use the *Explore* and *Graph* options of *IHT* to calculate and plot temperature histories at the cylinder center, $T(0, 0, t)$, and the midheight of the side, $T(r_o, 0, t)$, for $0 \le t \le 10$ min. Comment on the gradients occurring at these locations and what effect they might have on phase transformations and thermal stresses. *Hint*: In your sweep over the time variable, start at 1 s rather than zero.

(c) For $0 \le t \le 10$ min, calculate and plot temperature histories at the cylinder center, $T(0, 0, t)$, for convection coefficients of $500\ \text{W/m}^2\cdot\text{K}$ and $1000\ \text{W/m}^2\cdot\text{K}$.

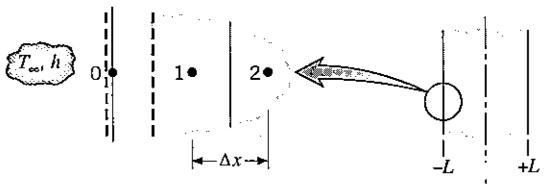
Finite-Difference Equations: Derivations

5.93 The stability criterion for the explicit method requires that the coefficient of the T_m^p term of the one-dimensional, finite-difference equation be zero or positive. Consider the situation for which the temperatures at the two neighboring nodes (T_{m-1}^p, T_{m+1}^p) are 100°C while the center node (T_m^p) is at 50°C. Show that for values of $Fo > \frac{1}{2}$ the finite-difference equation will predict a value of T_m^{p+1} that violates the second law of thermodynamics.

5.94 A thin rod of diameter D is initially in equilibrium with its surroundings, a large vacuum enclosure at temperature T_{sur}. Suddenly an electrical current I (A) is passed through the rod having an electrical resistivity ρ_e and emissivity ε. Other pertinent thermophysical properties are identified in the sketch. Derive the transient, finite-difference equation for node m.

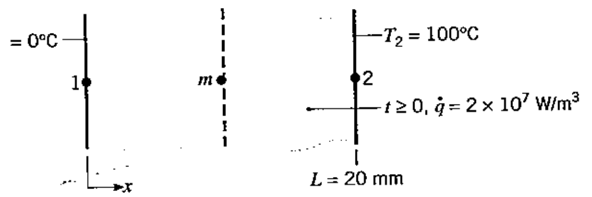


5.95 A one-dimensional slab of thickness $2L$ is initially at a uniform temperature T_i. Suddenly, electric current is passed through the slab causing a uniform volumetric heating $\dot{q}\ (\text{W/m}^3)$. At the same time, both outer surfaces ($x = \pm L$) are subjected to a convection process at T_∞ with a heat transfer coefficient h.



Write the finite-difference equation expressing conservation of energy for node 0 located on the outer surface at $x = -L$. Rearrange your equation and identify any important dimensionless coefficients.

96 A plane wall ($\rho = 4000\ \text{kg/m}^3$, $c_p = 500\ \text{J/kg}\cdot\text{K}$, $k = 10\ \text{W/m}\cdot\text{K}$) of thickness $L = 20$ mm initially has a linear, steady-state temperature distribution with boundaries maintained at $T_1 = 0°\text{C}$ and $T_2 = 100°\text{C}$. Suddenly, an electric current is passed through the wall, causing uniform energy generation at a rate $\dot{q} = 2 \times 10^7\ \text{W/m}^3$. The boundary conditions T_1 and T_2 remain fixed.



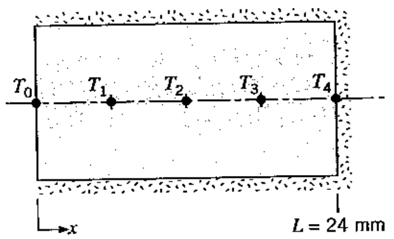
(a) On T–x coordinates, sketch temperature distributions for the following cases: (i) initial condition ($t \leq 0$); (ii) steady-state conditions ($t \to \infty$), assuming that the maximum temperature in the wall exceeds T_2; and (iii) for two intermediate times. Label all important features of the distributions.

(b) For the system of three nodal points shown schematically (1, m, 2), define an appropriate control volume for node m and, identifying all relevant processes, derive the corresponding finite-difference equation using either the *explicit* or *implicit* method.

$\boxed{\text{(c)}}$ With a time increment of $\Delta t = 5$ s, use the finite-difference method to obtain values of T_m for the first 45 s of elapsed time. Determine the corresponding heat fluxes at the boundaries, that is, q_x'' (0, 45 s) and q_x'' (20 mm, 45 s).

$\boxed{\text{(d)}}$ To determine the effect of mesh size, repeat your analysis using grids of 5 and 11 nodal points ($\Delta x = 5.0$ and 2.0 mm, respectively).

5.97 A round solid cylinder made of a plastic material ($\alpha = 6 \times 10^{-7}\ \text{m}^2/\text{s}$) is initially at a uniform temperature of 20°C and is well insulated along its lateral surface and at one end. At time $t = 0$, heat is applied to the left boundary causing T_0 to increase linearly with time at a rate of 1°C/s.



(a) Using the explicit method with $Fo = \frac{1}{2}$, derive the finite-difference equations for nodes 1, 2, 3, and 4.

(b) Format a table with headings of p, t (s), and the nodal temperatures T_0 to T_4. Determine the surface temperature T_0 when $T_4 = 35°\text{C}$.

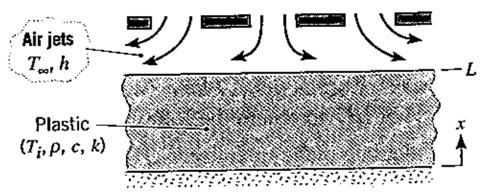
Finite-Difference Solutions: One-Dimensional Systems

$\boxed{5.98}$ A wall 0.12 m thick having a thermal diffusivity of $1.5 \times 10^{-6}\ \text{m}^2/\text{s}$ is initially at a uniform temperature of 85°C. Suddenly one face is lowered to a temperature of 20°C, while the other face is perfectly insulated.

(a) Using the explicit finite-difference technique with space and time increments of 30 mm and 300 s, respectively, determine the temperature distribution at $t = 45$ min.

(b) With $\Delta x = 30$ mm and $\Delta t = 300$ s, compute $T(x, t)$ for $0 \leq t \leq t_{ss}$, where t_{ss} is the time required for the temperature at each nodal point to reach a value that is within 1°C of the steady-state temperature. Repeat the foregoing calculations for $\Delta t = 75$ s. For each value of Δt, plot temperature histories for each face and the midplane.

5.99 A molded plastic product ($\rho = 1200\ \text{kg/m}^3$, $c = 1500\ \text{J/kg}\cdot\text{K}$, $k = 0.30\ \text{W/m}\cdot\text{K}$) is cooled by exposing one surface to an array of air jets, while the opposite surface is well insulated. The product may be approximated as a slab of thickness $L = 60$ mm, which is initially at a uniform temperature of $T_i = 80°\text{C}$. The air jets are at a temperature of $T_\infty = 20°\text{C}$ and provide a uniform convection coefficient of $h = 100\ \text{W/m}^2\cdot\text{K}$ at the cooled surface.



Using a finite-difference solution with a space increment of $\Delta x = 6$ mm, determine temperatures at the cooled and insulated surfaces after 1 hour of exposure to the gas jets.

$\boxed{5.100}$ The plane wall of Problem 2.47 ($k = 1.5\ \text{W/m}\cdot\text{K}$, $\alpha = 7.5 \times 10^{-6}\ \text{m}^2/\text{s}$) has a thickness of $L = 50$ mm and an initial uniform temperature of 25°C. Suddenly, the boundary at $x = L$ experiences heating by a fluid for which $T_\infty = 50°\text{C}$ and $h = 75\ \text{W/m}^2\cdot\text{K}$, while the other boundary at $x = 0$ experiences an applied heat flux of $q_o'' = 2000\ \text{W/m}^2$.

(a) With $\Delta x = 5$ mm and $\Delta t = 20$ s, compute and plot temperature distributions in the wall for (i) the initial condition, (ii) the steady-state condition, and (iii) two intermediate times.

(b) On q_x''–x coordinates, plot the heat flux distributions corresponding to the four temperature distributions represented in part (a).

(c) On q_x''–t coordinates, plot the heat flux at $x = 0$ and $x = L$.

$\boxed{5.101}$ The plane wall of Problem 2.48 ($k = 50\ \text{W/m}\cdot\text{K}$, $\alpha = 1.5 \times 10^{-6}\ \text{m}^2/\text{s}$) has a thickness of $L = 40$ mm and an initial uniform temperature of $T_o = 25°\text{C}$. Suddenly, the boundary at $x = L$ experiences heating by a fluid for which $T_\infty = 50°\text{C}$ and $h = 1000\ \text{W/m}^2\cdot\text{K}$, while heat is uniformly generated within the wall at $\dot{q} = 1 \times 10^7\ \text{W/m}^3$. The boundary at $x = 0$ remains at T_o.

(a) With $\Delta x = 4$ mm and $\Delta t = 1$ s, plot temperature distributions in the wall for (i) the initial condition, (ii) the steady-state condition, and (iii) two intermediate times.

(b) On q_x''–t coordinates, plot the heat flux at $x = 0$ and $x = L$. At what elapsed time is there zero heat flux at $x = L$?

5.102 Consider the fuel element of Example 5.8. Initially, the element is at a uniform temperature of 250°C with no heat generation. Suddenly, the element is inserted into the reactor core causing a uniform volumetric heat generation rate of $\dot{q} = 10^8\ \text{W/m}^3$. The surfaces are convectively cooled with $T_\infty = 250°\text{C}$ and $h = 1100\ \text{W/m}^2\cdot\text{K}$. Using the explicit method with a space increment of 2 mm, determine the temperature distribution 1.5 s after the element is inserted into the core.

$\boxed{5.103}$ Consider the fuel element of Example 5.8, which operates at a uniform volumetric generation rate of $\dot{q}_1 = 10^7\ \text{W/m}^3$, until the generation rate suddenly changes to $\dot{q}_2 = 2\times10^7\ \text{W/m}^3$. Use the *Finite-Difference Equations, One-Dimensional, Transient* conduction model builder of *IHT* to obtain the implicit form of the finite-difference equations for the 6 nodes, with $\Delta x = 2$ mm, as shown in the example.

(a) Calculate the temperature distribution 1.5 s after the change in operating power and compare your results with those tabulated in the example.

(b) Use the *Explore* and *Graph* options of *IHT* to calculate and plot temperature histories at the midplane (00) and surface (05) nodes for $0 \leq t \leq 400$ s. What are the steady-state temperatures, and approximately how long does it take to reach the new equilibrium condition after the step change in operating power?

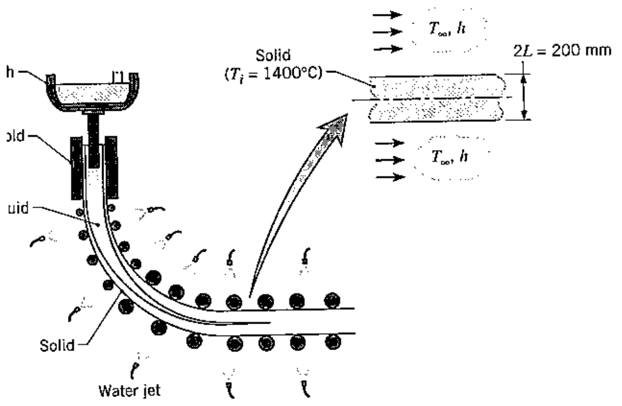
$\boxed{5.104}$ Consider the fuel element of Example 5.8, which operates at a uniform volumetric generation rate of $\dot{q}_1 = 10^7\ \text{W/m}^3$, until the generation rate suddenly changes to $\dot{q}_2 = 2\times10^7\ \text{W/m}^3$. Use the finite-element software *FEHT* to obtain the following solutions.

(a) Calculate the temperature distribution 1.5 s after the change in operating power and compare your results with those tabulated in the example. *Hint*: First determine the steady-state temperature distribution for $\dot{q}_1$, which represents the initial condition for the transient temperature distribution after the step change in power to $\dot{q}_2$. Next, in the *Setup* menu, click on *Transient*: in the *Specify/Internal Generation* box, change the value to $\dot{q}_2$; and in the *Run* command, click on *Continue* (not *Calculate*). See the *Run* menu in the *FEHT* Help section for background information on the *Continue* option.

(b) Use your *FEHT* model to plot temperature histories at the midplane and surface for $0 \leq t \leq 400$ s. What are the steady-state temperatures, and approximately how long does it take to reach the new equilibrium condition after the step change in operating power?

$\boxed{5.105}$ In a thin-slab, continuous casting process, molten steel leaves a mold with a thin solid shell, and the molten material solidifies as the slab is quenched by water jets on route to a section of rollers. Once fully solidified, the slab continues to cool as it is brought to an acceptable handling temperature. It is this portion of the process that is of interest.

Consider a 200-mm-thick solid slab of steel ($\rho = 7800\ \text{kg/m}^3$, $c = 700\ \text{J/kg}\cdot\text{K}$, $k = 30\ \text{W/m}\cdot\text{K}$), initially at a uniform temperature of $T_i = 1400°\text{C}$. The slab is cooled at its top and bottom surfaces by water jets ($T_\infty = 50°\text{C}$), which maintain an approximately



uniform convection coefficient of $h = 5000\ \mathrm{W/m^2 \cdot K}$ at both surfaces. Using a finite-difference solution with a space increment of $\Delta x = 1$ mm, determine the time required to cool the surface of the slab to 200°C. What is the corresponding temperature at the midplane of the slab? If the slab moves at a speed of $V = 15$ mm/s, what is the required length of the cooling section?

5.106 A very thick plate with thermal diffusivity $5.6 \times 10^{-6}\ \mathrm{m^2/s}$ and thermal conductivity 20 W/m · K is initially at a uniform temperature of 325°C. Suddenly, the surface is exposed to a coolant at 15°C for which the convection heat transfer coefficient is 100 W/m² · K. Using the finite-difference method with a space increment of $\Delta x = 15$ mm and a time increment of 18 s, determine temperatures at the surface and at a depth of 45 mm after 3 min have elapsed.

5.107 Referring to Example 5.9, Comment 4, consider a sudden exposure of the surface to large surroundings at an elevated temperature (T_{sur}) and to convection (T_∞, h).

(a) Derive the explicit, finite-difference equation for the surface node in terms of Fo, Bi, and Bi_r.

(b) Obtain the stability criterion for the surface node. Does this criterion change with time? Is the criterion more restrictive than that for an interior node?

(c) A thick slab of material ($k = 1.5$ W/m · K, $\alpha = 7 \times 10^{-7}\ \mathrm{m^2/s}$, $\varepsilon = 0.9$), initially at a uniform temperature of 27°C, is suddenly exposed to large surroundings at 1000 K. Neglecting convection and using a space increment of 10 mm, determine temperatures at the surface and 30 mm from the surface after an elapsed time of 1 min.

5.108 Consider the thick slab of copper in Example 5.9, which is initially at a uniform temperature of 20°C and is suddenly exposed to a net radiant flux of $3 \times 10^5\ \mathrm{W/m^2}$. Use the *Finite-Difference Equations/One-Dimensional/Transient* conduction model builder of *IHT* to obtain the implicit form of the finite-difference equations for the interior nodes. In your analysis, use a space increment of $\Delta x = 37.5$ mm with a total of 17 nodes (00–16), and a time increment of $\Delta t = 1.2$ s. For the surface node 00, use the finite-difference equation derived in Section 2 of the Example.

(a) Calculate the 00 and 04 nodal temperatures at $t = 120$ s, that is, $T(0, 120\ \mathrm{s})$ and $T(0.15\ \mathrm{m}, 120\ \mathrm{s})$, and compare the results with those given in Comment 1 for the exact solution. Will a time increment of 0.12 s provide more accurate results?

(b) Plot temperature histories for $x = 0$, 150, and 600 mm, and explain key features of your results.

5.109 Consider the thick slab of copper in Example 5.9, which is initially at a uniform temperature of 20°C and is suddenly exposed to large surroundings at 1000°C (instead of a prescribed heat flux).

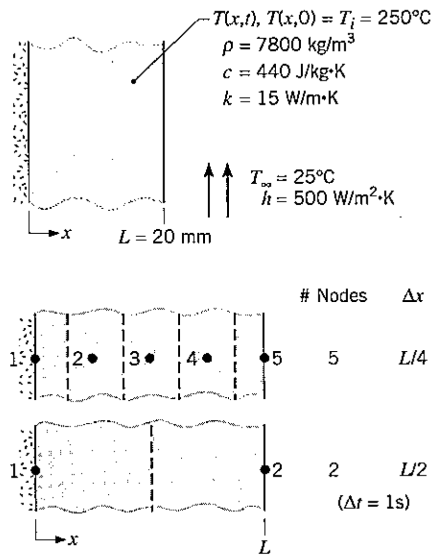
(a) For a surface emissivity of 0.94, calculate the temperatures $T(0, 120\ \mathrm{s})$ and $T(0.15\ \mathrm{m}, 120\ \mathrm{s})$ using the finite-element software *FEHT*. *Hint*: In the *Convection Coefficient* box of the *Specify/Boundary Conditions* menu of *FEHT*, enter the linearized radiation coefficient (see Equation 1.9) for the surface ($x = 0$). Enter the temperature of the surroundings in the *Fluid Temperature* box. See also the *Help* section on *Entering Equations*. Click on *Setup/Temperatures in K* to enter all temperatures in kelvins.

(b) Plot the temperature histories for $x = 0$, 150, and 600 mm, and explain key features of your results.

5.110 Consider the composite wall of Problem 2.53. In Part (d), you are asked to sketch the temperature histories at $x = 0, L$ during the transient period between Cases 2 and 3. Calculate and plot these histories using the finite-element method of *FEHT*, the finite-difference method of *IHT* (with $\Delta x = 5$ mm and $\Delta t = 1.2$ s), and/or an alternative procedure of your choice. If you use more than one method, compare the respective results. Note that, in using *FEHT* or *IHT*, a look-up table must be created for prescribing the variation of the heater flux with time (see the appropriate *Help* section for guidance).

5.111 In Section 5.5, the one-term approximation to the series solution for the temperature distribution was

developed for a plane wall of thickness $2L$ that is initially at a uniform temperature and suddenly subjected to convection heat transfer. If $Bi < 0.1$, the wall can be approximated as isothermal and represented as a lumped capacitance (Equation 5.7). For the conditions shown schematically, we wish to compare predictions based on the one-term approximation, the lumped capacitance method, and a finite-difference solution.

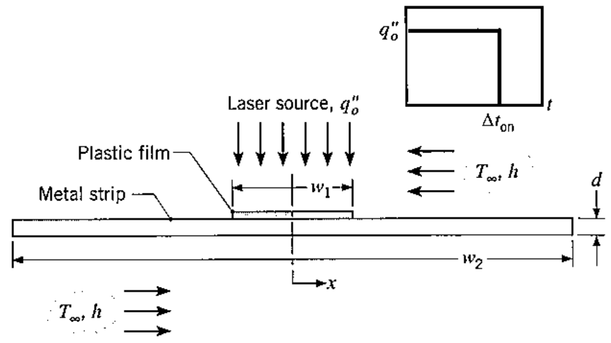


(a) Determine the midplane, $T(0, t)$, and surface, $T(L, t)$, temperatures at $t = 100$, 200, and 500 s using the one-term approximation to the series solution, Equation 5.40. What is the Biot number for the system?

(b) Treating the wall as a lumped capacitance, calculate the temperatures at $t = 50$, 100, 200, and 500 s. Did you expect these results to compare favorably with those from Part (a)? Why are the temperatures considerably higher?

(c) Consider the 2- and 5-node networks shown schematically. Write the implicit form of the finite-difference equations for each network, and determine the temperature distributions for $t = 50$, 100, 200, and 500 s using a time increment of $\Delta t = 1$ s. You may use *IHT* to solve the finite-difference equations by representing the rate of change of the nodal temperatures by the intrinsic function, Der(T, t). Prepare a table summarizing the results of Parts (a), (b), and (c). Comment on the relative differences of the predicted temperatures. *Hint*: See the *Solver/Intrinsic Functions* section of *IHT/Help* or the *IHT Examples* menu (Example 5.2) for guidance on using the Der(T, t) function.

5.112 Consider the bonding operation described in Problem 3.103, which was analyzed under steady-state conditions. In this case, however, the laser will be used to heat the film for a prescribed period of time, creating the transient heating situation shown in the sketch.



The strip is initially at 25°C and the laser provides a uniform flux of 85,000 W/m² over a time interval of $\Delta t_{on} = 10$ s. The system dimensions and thermophysical properties remain the same, but the convection coefficient to the ambient air at 25°C is now 100 W/m² · K and $w_1 = 44$ mm.

Using an implicit finite-difference method with $\Delta x = 4$ mm and $\Delta t = 1$ s, obtain temperature histories for $0 \le t \le 30$ s at the center and film edge, $T(0, t)$ and $T(w_1/2, t)$, respectively, to determine if the adhesive is satisfactorily cured above 90°C for 10 s and if its degradation temperature of 200°C is exceeded.

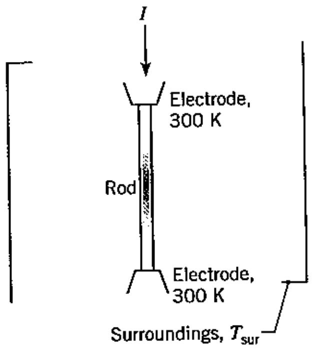
5.113 One end of a stainless steel (AISI 316) rod of diameter 10 mm and length 0.16 m is inserted into a fixture maintained at 200°C. The rod, covered with an insulating sleeve, reaches a uniform temperature throughout its length. When the sleeve is removed, the rod is subjected to ambient air at 25°C such that the convection heat transfer coefficient is 30 W/m² · K.

(a) Using the explicit finite-difference technique with a space increment of $\Delta x = 0.016$ m, estimate the time required for the midlength of the rod to reach 100°C.

(b) With $\Delta x = 0.016$ m and $\Delta t = 10$ s, compute $T(x, t)$ for $0 \le t \le t_1$, where t_1 is the time required for the midlength of the rod to reach 50°C. Plot the temperature distribution for $t = 0$, 200 s, 400 s, and t_1.

5.114 A tantalum rod of diameter 3 mm and length 120 mm is supported by two electrodes within a large vacuum enclosure. Initially the rod is in equilibrium with the electrodes and its surroundings, which are maintained at 300 K. Suddenly, an electrical current, $I = 80$ A, is passed through the rod.

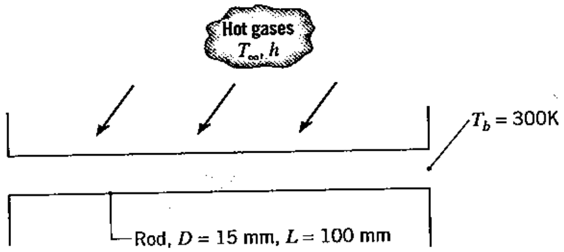
Assume the emissivity of the rod is 0.1 and the electrical resistivity is $95 \times 10^{-8}\ \Omega \cdot \text{m}$. Use Table A.1 to obtain the other thermophysical properties required in your solution. Use a finite-difference method with a space increment of 10 mm.



(a) Estimate the time required for the midlength of the rod to reach 1000 K.

(b) Determine the steady-state temperature distribution and estimate approximately how long it will take to reach this condition.

5.115 A support rod ($k = 15$ W/m · K, $\alpha = 4.0 \times 10^{-6}$ m²/s) of diameter $D = 15$ mm and length $L = 100$ mm spans a channel whose walls are maintained at a temperature of $T_b = 300$ K. Suddenly, the rod is exposed to a cross flow of hot gases for which $T_\infty = 600$ K and $h = 75$ W/m² · K. The channel walls are cooled and remain at 300 K.



(a) Using an appropriate numerical technique, determine the thermal response of the rod to the convective heating. Plot the midspan temperature as a function of elapsed time. Using an appropriate analytical model of the rod, determine the steady-state temperature distribution and compare the result with that obtained numerically for very long elapsed times.

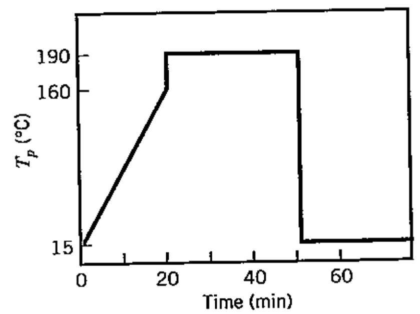
(b) After the rod has reached steady-state conditions, the flow of hot gases is suddenly terminated, and the rod cools by free convection to ambient air at $T_\infty = 300$ K and by radiation exchange with large surroundings at $T_{sur} = 300$ K. The free convection coefficient can be expressed as $h\ (\text{W/m}^2 \cdot \text{K}) = C\ \Delta T^n$, where $C = 4.4\ \text{W/m}^2 \cdot \text{K}^{1.188}$ and $n = 0.188$. The emissivity of the rod is 0.5. Determine the subsequent thermal response of the rod. Plot the midspan temperature as a function of cooling time, and determine the time required for the rod to reach a *safe-to-touch* temperature of 315 K.

5.116 Consider the acceleration-grid foil ($k = 40$ W/m · K, $\alpha = 3 \times 10^{-5}$ m²/s, $\varepsilon = 0.45$) of Problem 4.74. Develop an implicit, finite-difference model of the foil, which can be used for the following purposes.

(a) Assuming the foil to be at a uniform temperature of 300 K when the ion beam source is activated, obtain a plot of the midspan temperature–time history. At what elapsed time does this point on the foil reach a temperature within 1 K of the steady-state value?

(b) The foil is operating under steady-state conditions when, suddenly, the ion beam is deactivated. Obtain a plot of the subsequent midspan temperature–time history. How long does it take for the hottest point on the foil to cool to 315 K, a *safe-to-touch* condition?

5.117 Circuit boards are treated by heating a stack of them under high pressure as illustrated in Problem 5.42 and described further in Problem 5.43. A finite-difference method of solution is sought with two additional considerations. First, the book is to be treated as having distributed, rather than lumped, characteristics, by using a grid spacing of $\Delta x = 2.36$ mm with nodes at the center of the individual circuit board or plate. Second, rather than bringing the platens to 190°C in one sudden change, the heating schedule $T_p(t)$ shown below is to be used in order to minimize excessive thermal stresses induced by rapidly changing thermal gradients in the vicinity of the platens.

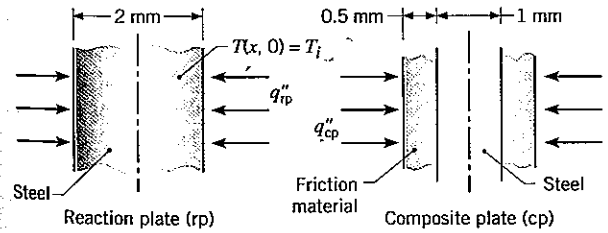


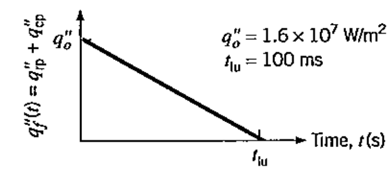
(a) Using a time increment of $\Delta t = 60$ s and the implicit method, find the temperature history of the midplane of the book and determine whether curing will occur (170°C for 5 min).

(b) Following the reduction of the platen temperatures to 15°C ($t = 50$ min), how long will it take for the midplane of the book to reach 37°C, a safe temperature at which the operator can begin unloading the press?

(c) Validate your program code by using the heating schedule of a sudden change of platen temperature from 15 to 190°C and compare results with those from an appropriate analytical solution (see Problem 5.43).

5.118 Common transmission failures result from the glazing of clutch surfaces by deposition of oil oxidation and decomposition products. Both the oxidation and decomposition processes depend on temperature histories of the surfaces. Because it is difficult to measure these surface temperatures during operation, it is useful to develop models to predict clutch-interface thermal behavior. The relative velocity between mating clutch plates, from the initial engagement to the zero-sliding (*lock-up*) condition, generates heat that is transferred to the plates. The relative velocity decreases at a constant rate during this period, producing a heat flux that is initially very large and decreases linearly with time, until lock-up occurs. Accordingly, $q_f'' = q_o''[1 - (t/t_{lu})]$, where $q_o'' = 1.6 \times 10^7$ W/m² and $t_{lu} = 100$ ms is the lock-up time. The plates have an initial uniform temperature of $T_i = 40$°C, when the prescribed frictional heat flux is suddenly applied to the surfaces. The reaction plate is fabricated from steel, while the composite plate has a thinner steel center section bonded to low-conductivity friction material layers. The thermophysical properties are $\rho_s = 7800$ kg/m³, $c_s = 500$ J/kg · K, and $k_s = 40$ W/m · K for the steel and $\rho_{fm} = 1150$ kg/m³, $c_{fm} = 1650$ J/kg · K, and $k_{fm} = 4$ W/m · K for the friction material.





(a) On $T - t$ coordinates, sketch the temperature history at the midplane of the reaction plate, at the interface between the clutch pair, and at the midplane of the composite plate. Identify key features.

(b) Perform an energy balance on the clutch pair over the time interval $\Delta t = t_{lu}$ to determine the steady-state temperature resulting from clutch engagement. Assume negligible heat transfer from the plates to the surroundings.

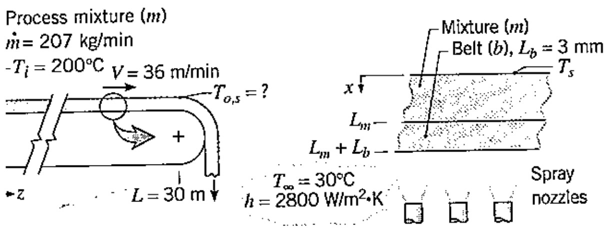
(c) Compute and plot the three temperature histories of interest using the finite-element method of *FEHT* or the finite-difference method of *IHT* (with $\Delta x = 0.1$ mm and $\Delta t = 1$ ms). Calculate and plot the frictional heat fluxes to the reaction and composite plates, q''_{rp} and q''_{cp}, respectively, as a function of time. Comment on features of the temperature and heat flux histories. Validate your model by comparing predictions with the results from Part (b). *Note*: Use of both *FEHT* and *IHT* requires creation of a look-up data table for prescribing the heat flux as a function of time.

5.119 Heat transfer is not an intuitive process muses the Curious Cook. Does doubling the thickness of a hamburger approximately double the cooking time? What effect does the initial temperature have on cooking time? To answer these questions, develop a model to do virtual cooking of meat of thickness $2L$ in a double-sided grill. The meat is initially at 20°C, when it is placed in the grill and both sides experience convection heat transfer characterized by an ambient temperature of 100°C and a convection coefficient of 5000 W/m² · K. Assume the meat to have the properties of liquid water at 300 K and to be properly cooked when the center temperature is 60°C.

(a) For hamburgers of thickness $2L = 10$, 20, and 30 mm, calculate the time for the center to reach the required cooking temperature of 60°C. Determine a relationship between the cooking time and the thickness. For your solution, use the finite-element method of *FEHT*, the ready-to-solve model in the *Models/Transient Conduction/Plane Wall* section of *IHT*, or a numerical procedure of your choice. For one of the thicknesses, use an appropriate analytical solution to validate your numerical results.

(b) Without performing a detailed numerical solution, but drawing on the results of Part (a), what can you say about the effect on the cooking time of changing the initial temperature of the meat from 20°C to 5°C? You may use your numerical model from Part (a) to confirm your assessment.

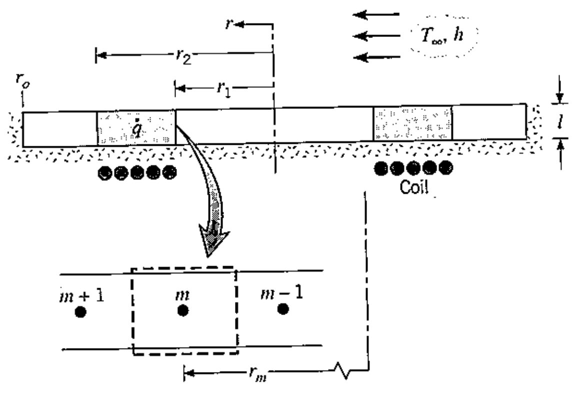
5.120 A process mixture at 200°C flows at a rate of 207 kg/min onto a conveyor belt of 3-mm thickness, 1-m width, and 30-m length traveling with a velocity of 36 m/min. The underside of the belt is cooled by a water spray at a temperature of 30°C, and the convection coefficient is 3000 W/m²·K. The thermophysical properties of the process mixture are $\rho_m = 960$ kg/m³, $c_m = 1700$ J/kg·K, and $k_m = 1.5$ W/m·K, while the properties for the conveyor (metallic) belt, are $\rho_b = 8000$ kg/m³, $c_b = 460$ J/kg·K, and $k_b = 15$ W/m·K.



Using the finite-difference method of *IHT* ($\Delta x = 0.5$ mm, $\Delta t = 0.05$ s), the finite-element method of *FEHT*, or a numerical procedure of your choice, calculate the surface temperature of the mixture at the end of the conveyor belt, $T_{o,s}$. Assume negligible heat transfer to the ambient air by convection or by radiation to the surroundings.

Finite-Difference Equations: Cylindrical Coordinates

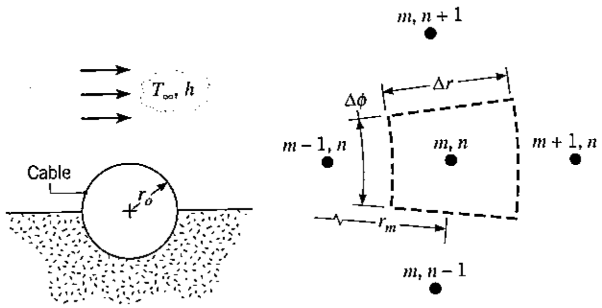
5.121 A thin circular disk is subjected to induction heating from a coil, the effect of which is to provide a uniform heat generation within a ring section as shown. Convection occurs at the upper surface, while the lower surface is well insulated.



(a) Derive the transient, finite-difference equation for node m, which is within the region subjected to induction heating.

(b) On T–r coordinates sketch, in a qualitative manner, the steady-state temperature distribution, identifying important features.

5.122 An electrical cable, experiencing a uniform volumetric generation $\dot{q}$, is half buried in an insulating material while the upper surface is exposed to a convection process (T_∞, h).



(a) Derive the explicit, finite-difference equations for an interior node (m, n), the center node $(m = 0)$, and the outer surface nodes (M, n) for the convection and insulated boundaries.

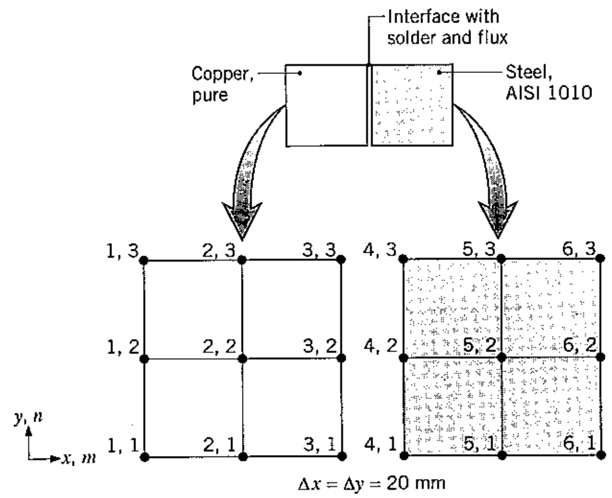
(b) Obtain the stability criterion for each of the finite-difference equations. Identify the most restrictive criterion.

Finite-Difference Solutions: Two-Dimensional Systems

5.123 Two very long (in the direction normal to the page) bars having the prescribed initial temperature distributions are to be soldered together. At time $t = 0$, the $m = 3$ face of the copper (pure) bar contacts the $m = 4$ face of the steel (AISI 1010) bar. The solder and flux act as an interfacial layer of negligible thickness and effective contact resistance $R''_{t,c} = 2 \times 10^{-5}$ m²·K/W.

Initial Temperatures (K)

n/m	1	2	3	4	5	6
1	700	700	700	1000	900	800
2	700	700	700	1000	900	800
3	700	700	700	1000	900	800



(a) Derive the explicit, finite-difference equation in terms of Fo and $Bi_c = \Delta x/kR''_{t,c}$ for $T_{4,2}$ and determine the corresponding stability criterion.

(b) Using $Fo = 0.01$, determine $T_{4,2}$ one time step after contact is made. What is Δt? Is the stability criterion satisfied?

5.124 Consider the stainless steel cylinder of Example 5.7, which is initially at 600 K and is suddenly quenched in an oil bath at 300 K, with $h = 500$ W/m²·K. Use the *ready-to-solve* model in the *Examples* menu of *FEHT* to obtain the following solutions.

(a) Calculate the temperatures, $T(r, x, t)$, after 3 min at the cylinder center, $T(0, 0, 3\text{ min})$, at the center of a circular face, $T(0, L, 3\text{ min})$, and the midheight of the side, $T(r_o, 0, 3\text{ min})$. Compare your results with those in the text example.

(b) Plot the temperature history at the center, $T(0, 0, t)$, and at the midheight of the side, $T(r_o, 0, t)$, for $0 \le t \le 10$ min using the *View/Temperatures vs. Time* command. Comment on the gradients occurring at these locations and what effect they might have on phase transformations and thermal stresses.

(c) Having solved the model for a total integration time of 10 min in part (b), now use the *View/Temperature Contours* command with the shaded band option for the isotherm contours. Select the *From Start to Stop* time option, and view the temperature contours as the cylinder cools during the quench process. Describe the major features of the cooling process revealed by this display. Use other options of this command to create a 10-isotherm temperature distribution for $t = 3$ min.

(d) For the location of Part (a), calculate the temperatures after 3 min if the convection coefficient is doubled ($h = 1000$ W/m²·K). Also, for convection coefficients of 500 and 1000 W/m²·K, determine how long the cylinder needs to remain in the oil bath to achieve a *safe-to-touch* surface temperature of 316 K. Tabulate and comment on the results of parts (a) and (d).

5.125 Consider the system of Problem 4.57. Initially with no flue gases flowing, the walls ($\alpha = 5.5 \times 10^{-7}$ m²/s) are at a uniform temperature of 25°C. Using the implicit, finite-difference method with a time increment of 1 h, find the temperature distribution in the wall 5, 10, 50, and 100 h after introduction of the flue gases.

5.126 Consider the system of Problem 4.75. Initially, the ceramic plate ($\alpha = 1.5 \times 10^{-6}$ m²/s) is at a uniform temperature of 30°C, and suddenly the electrical heating elements are energized. Using the implicit, finite-difference method, estimate the time required for the difference between the surface and initial temperatures to reach 95% of the difference for steady-state conditions. Use a time increment of 2 s.

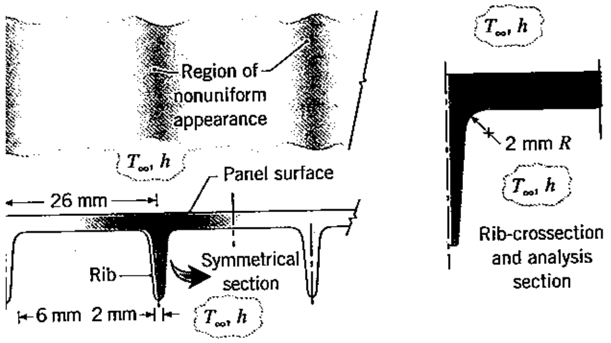
5.127 Consider the thermal conduction module and operating conditions of Problem 4.78. To evaluate the transient response of the cold plate, which has a thermal diffusivity of $\alpha = 75 \times 10^{-6}$ m²/s, assume that, when the module is activated at $t = 0$, the initial temperature of the cold plate is $T_i = 15$°C and a uniform heat flux of $q''_o = 10^5$ W/m² is applied at its base. Using the implicit finite-difference method and a time increment of $\Delta t = 0.1$ s, compute the designated nodal temperatures as a function of time. From the temperatures computed at a particular time, evaluate the ratio of the rate of heat transfer by convection to the water to the heat input at the base. Terminate the calculations when this ratio reaches 0.99. Print the temperature field at 5-s intervals and at the time for which the calculations are terminated.

5.128 The operations manager for a metals processing plant anticipates the need to repair a large furnace and has come to you for an estimate of the time required for the furnace interior to cool to a safe working temperature. The furnace is cubical with a 16-m interior dimension and 1-m thick walls for which $\rho = 2600$ kg/m³, $c = 960$ J/kg·K, and $k = 1$ W/m·K. The operating temperature of the furnace is 900°C, and the outer surface experiences convection with ambient air at 25°C and a convection coefficient of 20 $W/m^2 \cdot K$.

(a) Use a numerical procedure to estimate the time required for the inner surface of the furnace to cool to a safe working temperature of 35°C. *Hint*: Consider a two-dimensional cross-section of the furnace, and perform your analysis on the smallest symmetrical section.

(b) Anxious to reduce the furnace downtime, the operations manager also wants to know what effect circulating ambient air through the furnace would have on the cool-down period. Assume equivalent convection conditions for the inner and outer surfaces.

.129 The door panel of an automobile is fabricated by a plastic hot-extrusion process resulting in the ribbed cross section shown schematically. Following a process involving air-cooling, painting, and baking, the panel is ready for assembly on the vehicle. However, upon visual inspection, the rib pattern is evident on the outer surface. In regions over the ribs, the paint has an "orange peel" appearance, making the door panel unacceptable for use. The apparent reason for this defect is the variable microfinish caused by differential cooling rates at the surface of the panel, which affect adherence of the paint. Your assignment is to estimate the panel surface temperature distribution as a function of cooling time.



Use the finite-element software *FEHT* to obtain the temperature distribution in the symmetrical section shown schematically. The panel is ejected from the extrusion press at a uniform temperature of 275°C and is allowed to cool on a transport table, where the air temperature is 25°C and the convection coefficient is 10 $W/m^2 \cdot K$. The thermophysical properties of the extruded plastic are $\rho = 1050\ kg/m^3$, $c = 800\ J/kg \cdot K$, and $k = 0.5\ W/m \cdot K$.

(a) Using the *View/Temperature vs. Time* command, plot temperature histories for selected locations on the panel surface. Is there noticeable differential cooling in the region above the rib? If so, what is the cause? *Hint*: When you draw the shape outline in *FEHT*, represent the fillet region by six or seven segments that approximate the fillet radius. Then draw element lines to form triangular elements.

(b) Use the *View/Temperature Contours* command with the shaded band option to plot the isotherm contours. Select the *From Start to Stop* time option, and view the temperature contours as the panel cools. Describe major features of the cooling process observed from the display. Use other options of this command to create a 10-isotherm temperature distribution for a time that illustrates some of the foregoing important features. How would you redesign the ribbed panel to reduce this thermally induced defect, while still retaining the stiffening function required of the ribs?

CHAPTER **6**

Introduction to Convection